EAMCET-TS 2017

ENGINEERING

Question Paper with Solutions

CODE-A

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MATHS

1. **If tan** $20^0 = \lambda,$ **then** $\dfrac{\tan 160^0 - \tan 110^0}{1+(\tan 160^0)(\tan 110^0)} =$

1) $\dfrac{1+\lambda^2}{2\lambda}$ 2) $\dfrac{1+\lambda^2}{\lambda}$ 3) $\dfrac{1-\lambda^2}{\lambda}$ 4) $\dfrac{1-\lambda^2}{2\lambda}$

Key: 4

Sol: . $\dfrac{\tan 20 + \cot 20}{1+1} = \dfrac{-\lambda + \frac{1}{\lambda}}{2} = \dfrac{1-\lambda^2}{2\lambda}$

2 . **Consider the circle** $x^2+y^2-6x+4y=12$ **.The equation of a tangent to this circle that is parallel to the line** $4x+3y+5=0$ **is**

1) $4x+3y+10=0$ 2) $4x+3y-9=0$ 3) $4x+3y+9=0$ 4) $4x+3y-31=0$

Key: 4

Sol: . $y+f = m(x+g) \pm r\sqrt{1+m^2}$

$$= y+2 = \frac{-4}{3}(x-3) \pm 5\sqrt{1+\frac{16}{9}}$$

$$y+2 = \frac{-4}{3}\left(x-3\right) \pm \frac{25}{3}$$

$$3y+6 = -4x+12 \pm 25$$

$$4x+3y-6 \pm 25$$

$$4x+3y+19=0$$

3. **The mean deviation from the mean 10 of the date 6,7,10,12,13,** α **,12,16 is**

1) 3.5 2) 3.25 3) 3 4) 3.75

Key: 2

Sol: . $\bar{x} = \dfrac{6+7+10+12+13+\alpha+17+16}{8} = 10$

$\Rightarrow \alpha = 4$; M.D= $\dfrac{\sum\left|x_i - \bar{x}\right|}{n} = 3.25$

4. **Match the following**

List -I	**List II**
I. $\int_{-1}^{1} x\left|x\right| dx$	**a)** $\dfrac{\pi}{2}$
II. $\int_0^{\frac{\pi}{2}} \left(1+\log\left(\dfrac{4+3\sin x}{4+3\cos x}\right)\right) dx$	**b)** $\int_0^{\frac{a}{2}} f(x)dx$
III. $\int_0^a f(x)dx$	**c)** $\int_0^a \left[f(x)+f(-x)\right]dx$
IV. $\int_{-a}^{a} f(x)dx$	**d)** 0
	e) $\int_0^a f(a-x)dx$

	I	II	III	IV
1)	d	a	e	c
2)	d	a	c	b
3)	d	c	a	e
4)	a	d	b	c

Key: 1

Sol: .Conceptual

5. **If f is differentiable ,** $f(x+y)=f(x)f(y)$ **for all** $x,y\in R, f(3)=3, f^{'}(0)=11,$ **then** $f^{'}(3)=$

1) $\frac{3}{11}$ 2) $\frac{11}{3}$ 3) 8 4) 33

Key: 4

Sol: . $f^{'}(0)f(3)=3(11)=33$

6. $\int_0^{\pi}\frac{xdx}{4\cos^2 x+9\sin^2 x}=$

1) $\frac{\pi^2}{12}$ 2) $\frac{\pi^2}{4}$ 3) $\frac{\pi^2}{6}$ 4) $\frac{\pi^2}{3}$

Key: 1

Sol: .G.I = $\frac{\pi^2}{2ab}$

7. **The probability distribution of a random variable X is given below**

X=k	**0**	**1**	**2**	**3**	**4**
P(X=k)	**0.1**	**0.4**	**0.3**	**0.2**	**0**

The variance of X is

1) 1.6 2) 0.24 3) 0.84 4) 0.75

Key: 3

Sol: Let. $\overline{\mu}=0+0.4+0.6+0.6+0=1.6$

$\sigma^2=0+0.4+1.2+1.8+0-(1.6)^2=0.84$

8. **If** $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 2 & 0\\ 1 & -1 & 4\end{bmatrix}, A=B+C, B=B^T$ **and** $C=-C^T$**, then C=**

1) $\begin{bmatrix}0 & 0.5 & 0\\ -0.5 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ 2) $\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0.5\\ 0 & -0.5 & 0\end{bmatrix}$ 3) $\begin{bmatrix}0 & -0.5 & 0.5\\ 0.5 & 0 & 0\\ -0.5 & 0 & 0\end{bmatrix}$ 4) $\begin{bmatrix}0 & 0.5 & 0\\ -0.5 & 0 & 0.5\\ 0 & -0.5 & 0\end{bmatrix}$

Key: 2

Sol: . $C=\frac{A-A^T}{2}$

9. **If** $\vec{a}$ **is a unit vector, then** $\left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{j}\right|^2+\left|\vec{a}\times\hat{k}\right|^2=$

1) 2 2) 4 3) 1 4) 0

Key: 1

Sol: .$\left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{j}\right|^2+\left|\vec{a}\times\hat{k}\right|^2=2\left|\vec{a}\right|^2=2$

10. A bag contains 5 red balls, 3 black balls and 4 white balls. Three balls are drawn at random. The probability that they are not of same colour is

1) $\frac{37}{44}$ 2) $\frac{31}{44}$ 3) $\frac{21}{44}$ 4) $\frac{41}{44}$

Key: 4

Sol: .Required probability = $1-\left[\frac{5_{C_3}+3_{C_3}+4_{C_3}}{12_{C_3}}\right]$

= $\frac{41}{44}$

11. The radical centre of the circles

$x^2+y^2-4x-6y+5=0, x^2+y^2-2x-4y-1=0, x^2+y^2-6x-2y=0$ **lies on the line**

1) $x+y-5=0$ 2) $2x-4y+7=0$ 3) $4x-6y+5=0$ 4) $18x-12y+1=0$

Key: 4

Sol: .Equations of radical axies of (1) and (2) and (2) and (3) are

$x+y-3=0, 4x-2y-1=0$ by solving these two equations weget R.C= $\left(\frac{7}{6},\frac{11}{6}\right)$

by verification R.C lies on 18x-12y+1=0

12. If $\cos ec\theta-\cot\theta=2017,$ **then quadrant in which** θ **lies is**

1) I 2) IV 3) III 4) II

Key: 4

Sol: . $\cos ec\theta-\cot\theta=2017$ .... (1)

$\cos ec\theta+\cot\theta=\frac{1}{2017}$ .... (2)

then $\cos ec\theta>0\,\&\cot\theta<0$

$\Rightarrow\theta\in Q_2$

13. If $\int e^{2x}f^{'}(x)dx=g(x),$ **then** $\int\left(e^{2x}f(x)+e^{2x}f^{'}(x)\right)dx=$

1) $\frac{1}{2}\left[e^{2x}f(x)-g(x)\right]+C$ 2) $\frac{1}{2}\left[e^{2x}f(x)+g(x)\right]+C$

3) $\frac{1}{2}\left[e^{2x}f(2x)+g(x)\right]+C$ 4) $\frac{1}{2}\left[e^{2x}f^{'}(x)+g(x)\right]+C$

Key: 2

Sol: .G.I = $\int e^{2x}f(x)dx+\int e^{2x}f^{'}(x)dx$

$=\int e^{2x}f(x)dx+g(x)$

$f(x)\int e^{2x}dx-\int f^{'}(x)\frac{e^{2x}}{2}dx+g(x)$

$$f(x)\frac{e^{2x}}{2}-\frac{1}{2}\int e^{2x}f^{'}(x)dx+g(x)$$

$$f(x)\frac{e^{2x}}{2}+\frac{1}{2}g(x)$$

$$=\frac{1}{2}\left[f(x)e^{2x}+g(x)\right]+C$$

14. **If $A=(5,3),B(3,-2)$ and a point P is such that the area of the triangle PAB is 9, then the locus of P represents**

1) a circle	2) a pair of coincident lines
3) a pair of parallel lines	4) a pair of perpendicular lines

Key: 3

Sol: .Area of $\Delta^{le}PAB=9$

$$\Rightarrow(5x-2y-37)(5x-2y-1)=0$$

then locus represents a pair of parallel lines

15. **A straight line makes an intercept on the Y-axis twice as long as that on X-axis and is at a unit distance from the torigin. Then the lines is represennted by the equations**

1) $2x+3y=\pm\sqrt{5}$	2) $x+y=\pm 2$	3) $x-y=\pm 2$	4) $2x+y=\pm\sqrt{5}$

Key: 4

Sol: .Let the equation be $\frac{x}{a}+\frac{y}{2a}=1$

given $\perp^{lr}$ distance = 1

$$\Rightarrow\frac{|2a|}{\sqrt{5}}=1$$

$$\Rightarrow 2a=\pm\sqrt{5}$$

$\therefore$ required line is $2x+y=\pm\sqrt{5}$

16. **Let S and $S^{'}$ be the foci of an ellipse and B be one end of its minor axis. If $SBS^{'}$ is an isosceles right angled triangle then the eccentricity of the ellipse is**

1) $\frac{1}{\sqrt{2}}$	2) $\frac{1}{2}$	3) $\frac{\sqrt{3}}{2}$	4) $\frac{1}{3}$

Key: 1

Sol: .$Tan\,45=\frac{b}{ae}$

$$\Rightarrow e=\frac{1}{\sqrt{2}}$$

17. **For the parabola** $y^2+6y-2x=-5$

I) the vertex is (-2,-3) **II) the directrix is y+3=0**

Which of the following is correct?

1) Both I and II are correct 2) I is true , II is false

3) Both I and II are false 4) I is false, II is true

Key: 2

Sol: .Given parabola rediuced in the form of $(y+3)^2=2(x+2)$

vertex = -2,-3

equation of directrix is $x-h+a=0$

18. **If** $\frac{x^2+5}{(x^2+1)(x-2)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1}$, **then A+B+C=**

1) -1 2) $\frac{2}{5}$ 3) $\frac{-3}{5}$ 4) 0

Key: 3

Sol: . $x^2+5=A(x^2+1)+(Bx+C)(x-2)$

by comparing like term on both sides then weget

$A=\frac{9}{5}, B=\frac{-4}{5}, C=\frac{-8}{5}$

19. $(x+iy)(1-2i)$ **is** $(1+i)$, **then**

1) $x+iy=1-i$ 2) $x+iy=\frac{1-i}{1-2i}$ 3) $x-iy=\frac{1-i}{1+2i}$ 4) $x-iy=\frac{1-i}{1+i}$

Key: 2

Sol: .Given $(x+iy)(1-2i)=1+i$

by comparing real and imaginary parts we get

$\Rightarrow x+2y=1\ \&\ 2x-y=1$

by solving these equations weget $x=\frac{3}{5}, y=\frac{1}{5}$

by verification $x+iy=\frac{1-i}{1-2i}$

$x+iy=\frac{1-i}{1-2i}\times\frac{1+2i}{1+2i}$

$=\frac{3+i}{5}$

20. $\int x^4e^{2x}dx=$

1) $\frac{e^{2x}}{4}(2x^4-4x^3+6x^2-6x+3)+C$ 2) $\frac{e^{2x}}{2}(2x^4-4x^3+6x^2-6x+3)+C$

3) $\frac{e^{2x}}{8}(2x^4+4x^3+6x^2+6x+3)+C$ 4) $-\frac{e^{2x}}{4}(2x^4+4x^3+6x^2+6x+3)+C$

Key: 1

Sol: .G.I = $x^4\left(\frac{e^{2x}}{2}\right)-4x^3\left(\frac{e^{2x}}{4}\right)+12x^2\left(\frac{e^{2x}}{8}\right)-24x\left(\frac{e^{2x}}{16}\right)+24\left(\frac{e^{2x}}{32}\right)+C$

$$\frac{e^{2x}}{4}(2x^4-4x^3+6x^2-6x+3)+C$$

21. **The sides of a triangle are in the ratio $1:\sqrt{3}:2$. Then the angles are in the ratio**

1) 1:2:3
2) 1:2:4
3) 1:4:5
4) 1:3:5

Key: 1

Sol: .Given $a:b:c=1:\sqrt{3}:2$

$$\frac{a}{1}=\frac{b}{\sqrt{3}}=\frac{c}{2}=k$$

by using the formulas $\cos A, CosB, CosC$ we get $A:B:C$
=1:2:3

22. **The sum of the complex roots of the equation $(x-1)^3+64=0$ is**

1) 6
2) 3
4) 6i
4) 3i

Key: 2

Sol: . $x-1=4(-1)^{1/3}=4(-1,-\omega,-\omega^2)$

$$=-4,-4\omega,-4\omega^2$$

$$x-1=-4,-4\omega,-4\omega^2$$

$$x=-3,1-4\omega,1-4\omega^2$$

sum of complex roots is = $1-4\omega+1-4\omega^2=6$

23. **The area of the region bounded by the curves $x=y^2-2$ and x=y is**

1) $\frac{9}{4}$
2) 9
3) $\frac{9}{2}$
4) $\frac{9}{7}$

Key: 3

Sol: .By solving given equations weget $y=-1, y=2$

required area = $\int_{-1}^{2}\left|y^2-2-y\right|dy=\frac{9}{2}$

24. **If $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$, then $\left(\vec{a}\times\hat{i}\right).\left(\hat{i}+\hat{j}\right)+\left(\vec{a}\times\hat{j}\right).\left(\hat{j}+\hat{k}\right)+\left(\vec{a}\times\hat{k}\right).\left(\hat{k}+\hat{i}\right)$**

1) $x-y+z$
2) $x+y+z$
3) $x+y-z$
4) $-x+y+z$

Key: 2

Sol: . $\bar{a}\times\bar{i}=\left(x\bar{k}+z\bar{k}\right)$

$$\bar{a}\times\bar{j}=\left(x\bar{k}-z\bar{k}\right)$$

$$\bar{a}\times\bar{k}=\left(-x\bar{j}+y\bar{i}\right)$$

then we get x+y+z

25. If the imaginary part of $\frac{2z+1}{iz+1}$ is -2, then the locus of the point representing z in the complex plane is

1) a circle 2) a parabola 3) a straight line 4) an ellipse

Key: 3

Sol: $\frac{2z+1}{1z+1}=\frac{2(x+iy)+1}{i(x+iy)+1}$

by simplyfing we get the equation $x+2y-2=0$ it repregents a straight line.

26. Let $f:(-1,1)\to R$ be a differentiable function with $f(0)=-1$ and $f^{'}(0)=1$. If $g(x)=(f(2f(x)+2))^2$, then $g^{'}(0)=$

1) 0 2) -2 3) 4 4) -4

Key: 4

Sol: $g(x)=f\left[2f(x)+2\right]^2$

$g^1(x)=2f\left[2f(x)+2\right]\ f^1\left[2f(x)+2\right]\times 2f^1(x)$

Now $g^1(0)=2f\left[2f(0)+2\right]\times f^1\left[2f(0)+2\right]\times 2f^1(0)$

$=-4$

27. If the perpendicular distance between the point (1,1) to the line $3x+4y+c=0$ is 7, then the possible values of c are

1) -35,42 2) 35,28 3) 42,-28 4) 28,-42

Key: 4

Sol: $\frac{|3+4+c|}{\sqrt{9+16}}=7$

$\Rightarrow |7+c|=35$

$\Rightarrow c=28, c=-42$

28. The solution of $\frac{dy}{dx}=\frac{x+y}{x-y}$ is

1) $\tan^{-1}\left(\frac{x}{y}\right)=\log\sqrt{x^2+y^2}+C$ 2) $\tan^{-1}\left(\frac{y}{x}\right)=\log\sqrt{x^2-y^2}+C$

3) $\sin^{-1}\left(\frac{y}{x}\right)=\log\sqrt{x^2+y^2}+C$ 4) $\cos^{-1}\left(\frac{y}{x}\right)=\log\sqrt{x^2-y^2}+C$

Key: 1

Sol: .put y=vx then given equation becomes $x\frac{dv}{dx}=\frac{1+v^2}{1-v}$

by separating variables by apply integrationn then we get

$\tan^{-1}\left(\frac{y}{x}\right)=\log\left(\sqrt{x^2+y^2}\right)+c$

29. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\frac{d^2y}{dx^2}=$

1) $\frac{-b^4}{a^2y^3}$ 2) $\frac{b^2}{ay^2}$ 3) $\frac{-b^3}{a^2y^3}$ 4) $\frac{b^3}{a^2y^2}$

Key: 1

Sol: .diff w.r to ‘x’ on both sides

$$\frac{dy}{dx}=\frac{-b^2x}{a^2y}$$

again diff w.r to ‘x’ on both sides

$$\frac{d^2y}{dx^2}=-\frac{b^4}{a^2y^3}$$

30. $\lim_{y\to1}\left(\frac{1}{y^2-1}-\frac{2}{y^4-1}\right)=$

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) 0

Key: 1

Sol: . $\lim_{y\to1}\frac{1}{y^2-1}-\frac{2}{\left(y^2-1\right)\left(y^2+1\right)}$

31. The solution of $(y-3x^2)dx+xdy=0$ is

1) $y(x)=\sin x+\frac{1}{x^2}+C$ 2) $y(x)=\cos x+\frac{1}{x^2}+C$

3) $y(x)=x^2+\frac{C}{x}$ 4) $y(x)=\sqrt{x}+\frac{C}{x}$

Key: 3

Sol: . $ydx+xdy=3x^2dx$

$$d(xy)=3x^2dx$$

integrating on both sides then we get $y=x^2+\frac{c}{x}$

32. If the coefficients of $(2r+1)^{th}$ term and $(r+1)^{th}$ term in the expansion of $(1+x)^{42}$ are equal then r can be

1) 12 2) 14 3) 16 4) 20

Key: 2

Sol: .Given that $T_{2r+1}=T_{r+1}$ then weget r=14

33. A point on the plane that passes through the points $(1,-1,6),(0,0,7)$ and perpendicular to the plane x-2y+z=6 is

1) $(1,-1,2)$ 2) $(1,1,2)$ 3) $(-1,1,2)$ 4) $(1,1,-2)$

Key: 2

Sol: Let the plane passing through poit $(1,-1,6)$ is

$a(x-1)+b(y+1)+c(z-6)=0$ ....(1)

If this plae pssing through $(0,0,7)$

$\Rightarrow -a+b+c=0$ ....(2)

If the plane $\perp r$ to the given plane

Then $a-2b+c=0$ ....(3)

From (2) & (3)

We get equation of plane is $3x+2y+z-7=0$

By verification (2) option satisifying the plane equation.

34. If the slope of the tangent to the curve $y=ax^3+bx+4$ at (2,14) is 21, then the values of a and b are respectively

1) 2,-3 2) 3,-2 3) -3,-2 4) 2,3

Key: 1

Sol: . $\frac{dy}{dx}=3ax^2+b$

slope (m)= 12a+b

$\Rightarrow 12a+b=21$ .... (1)

sub point on the curve then weget 4a+b=5 ....(2)

by solving (1) and (2) weget a=2,b=-3

35. The probability distribution of a random variable X is given below.

x	**1**	**2**	**3**	**4**	**5**	**6**
P(X=x)	**a**	**a**	**a**	**b**	**b**	**0.3**

If mean of X is 4.2, then a and b are respectivley equal to

1) 0.3,0.2 2) 0.1,0.4 3) 0.1,0.2 4) 0.2,0.1

Key: 3

Sol: .sum of prbabilities =1

$\Rightarrow 3a+2b=0.7$ ....(1)

given mean=4.2

$\Rightarrow a+2a+3a+4b+5b+1.8=4.2$

$\Rightarrow 6a+9b=2.4$ .... (2)

by solving 1 and 2 we get a=0.1, b=0.2

36. Let f(x) be a quadratic expression such that f(0)+f(1)=0. If f(-2)=0, then

1) $f\left(\frac{-2}{5}\right)=0$ 2) $f\left(\frac{2}{5}\right)=0$ 3) $f\left(\frac{-3}{5}\right)=0$ 4) $f\left(\frac{3}{5}\right)=0$

Key: 4

Sol: . $f(0)+f(1)=0 \Rightarrow a+b+2c=0$ ....(1)

$f(-2)=0 \Rightarrow 4a-2b+c=0$ .... (2)

by soving (1) and (2)

$\frac{a}{5}=\frac{b}{7}=\frac{c}{-6}$

then f(x)= $5x^2+7x-6$

by verification $f\left(\frac{3}{5}\right)=0$

37. The equation of tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ **at the point (a,b) is**

1) $\frac{x}{a}=-\frac{y}{b}$ 2) $\frac{x}{a}+\frac{y}{b}=2$ 3) $\frac{x}{a}=\frac{y}{b}$ 4) $\frac{x}{a}+\frac{y}{b}=n$

Key: 2

Sol: . diff w.r to 'x' on both sides then slope (m)= $-\frac{b}{a}$

then equation of tangent is $y-y_1=m(x-x_1)$

$$\Rightarrow\frac{x}{a}+\frac{y}{b}=2$$

38. If the line $x+y+k=0$ **is a normal to the hyperbola** $\frac{x^2}{9}-\frac{y^2}{4}=1$ **then k=**

1) $\pm\frac{\sqrt{5}}{13}$ 2) $\pm\frac{13}{\sqrt{5}}$ 3) $\pm\frac{13}{5}$ 4) $\pm\frac{5}{13}$

Key: 2

Sol: Normal condition is $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{\left(a^2+b^2\right)^2}{n^2}$

by sub the values we get k= $\pm\frac{13}{\sqrt{5}}$

39. The product of the real roots of $x^2-8x+9-\frac{8}{x}+\frac{1}{x^2}=0$ **is**

1) 2 2) 1 3) 3 4) 7

Key: 2

Sol: .By solving given equation weget

$x^4-8x^3+9x^2-8x+1=0$

$S_4=\alpha\beta\gamma\delta=1$

40. If $\Delta=\begin{vmatrix}1&5&6\\0&1&7\\0&0&1\end{vmatrix}$ **and** $\Delta^{'}=\begin{vmatrix}1&0&1\\3&0&3\\4&6&100\end{vmatrix}$**, then**

1) $\Delta^2-3\Delta^1=0$ 2) $\left(\Delta+\Delta^1\right)^2-3\left(\Delta+\Delta^1\right)+2=0$

3) $\left(\Delta+\Delta^1\right)^2+3\left(\Delta+\Delta^1\right)+5=0$ 4) $\Delta+3\Delta^1+1=0$

Key: 2

Sol: . $\Delta=1,\Delta^{'}=0$ satisfies the second option

41. A village has 10 players. A team of 6 players is to be formed. 5 members are chosen first out of thes 10 players and then captain is chosen from the remaining players. Then the total number of ways of cho9osing such teams is

1) 1260 2) 210 3) $(10C_6)5!$ 4) $(10C_5)6$

Key: 1

Sol: ${}^{10}C_5 \times {}^{5}C_1 = 1260$

42. The equation of the straiht line passing throught the point of intersection of $5x-6y-1=0, 3x+2y+5=0$ **and perpendicular to the line** $3x-5y+11=0$ **is**

1) $5x+3y+18=0$ 2) $-5x-3y+18=0$ 3) $5x+3y+8=0$ 4) $5x+3y-8=0$

Key: 3

Sol: $5x-6y-1=0$

$3x+2y+5=0$

$-6 \ \ -1 \ \ 5 \ \ -6$

$2 \ \ \ \ 5 \ \ 3 \ \ \ 2$

$$\frac{x}{-30+2} = \frac{y}{-3-25} = \frac{1}{10+18}$$

$x=-1, \quad y=-1$

$p=(-1,-1)$

$3x-5y+11=, m=\frac{3}{5}$

$y+1=\frac{-5}{3}(n+1)$

$3y+3=-5x-5$

$5x+3y+8=0$.

43. An integer is choosen from $\{2k/-9 \le k \le 10\}$. **The probability that it is divisible by both 4 and 6 is**

1) $\frac{1}{10}$ 2) $\frac{1}{20}$ 3) $\frac{1}{4}$ 4) $\frac{3}{20}$

Key: 4

Sol: $s=\{-18,-16,-14,-12,0,12,14,16,18,20\}$

$n(s)={}^{20}C_1$, the numbers divisible by both 4 and $6=n(E)=3$

$p(E)=\frac{3}{20}$.

44. $\int \frac{dx}{x\left(x^4+1\right)} =$

1) $\frac{1}{4}\log\left(\frac{x^4+1}{x^4}\right)+C$ 2) $\frac{1}{4}\log\left(\frac{x^4}{x^4+1}\right)+C$ 3) $\frac{1}{4}\log\left(x^4+1\right)+C$ 4) $\frac{1}{4}\log\left(\frac{x^4}{x^4+2}\right)+C$

Key: 2

Sol: $\int \frac{1}{x\left(x^4+1\right)}dx$

$\int\left(\frac{1}{x}-\frac{3}{x^4+1}\right)$

$\int \frac{1}{x}dx-\frac{1}{4}\int \frac{4x^3}{x^y+1}dx$

$=\cos|x|-\frac{1}{4}\left|1+x^4\right|+c$

$=\frac{1}{4}\cos|x|^4-\frac{1}{4}\left|1+x^4\right|+c$

$=\frac{1}{4}les\left|\frac{x^4}{1+x^4}\right|.$

45. $\sin^{-1}\frac{\sqrt{3}}{2}+\sin^{-1}\sqrt{\frac{2}{3}}=$

1) $\sin^{-1}\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}$

2) $\pi-\sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$

3) $-\pi-\sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$

4) $\pi+\sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$

Key: 2

Sol: $\sin\frac{\sqrt{3}}{2}+\sin\sqrt{\frac{2}{3}}$

$x=\frac{\sqrt{3}}{2}, y=\frac{\sqrt{2}}{3}$

$x^2+y^2>1$

$\sin^{-1}x+\sin^{-1}y=\pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$

$=\pi-\sin^{-1}$

$\pi-\sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$

46. α **and** β **are the roots of** $x^2+2x+C=0.$ **If** $\alpha^3+\beta^3=4,$ **then the value of C is**

1) -2 2) 3 3) 2 4) 4

Key: 3

Sol: $\alpha+\beta=-2, \alpha\beta=C, \left|\alpha^3+\beta^3=4\right|$

$\alpha^3+\beta^3=\left(\alpha+\beta\right)^3-3\alpha\beta\left(\alpha+\beta\right)$

$4=-8-3c\left(-2\right)$

$4=-8+6c$

$\Rightarrow 6c=12\Rightarrow c=2\cdot$

47. If the slope of the tangent to the circle $S\equiv x^2+y^2-13=0$ at (2,3) is m, then the point $\left(m,\dfrac{-1}{m}\right)$ is

1) an external point with respect to the circle $S=0$

2) an internal point with respect to the circle $S=0$

3) the centre of the circle $S=0$ 4) a point on the circle $S=0$

Key: 2

Sol: Given ole $x^2+y^2-13=0$

Eqn of tangent at $\left(2,3\right)$

$2x+3y-13=0$

slope $m=\dfrac{-2}{3}$

$p\left(m,\dfrac{-1}{m}\right)=\left(\dfrac{-2}{3},\dfrac{3}{2}\right)$

$\therefore s_{11}<0\ \therefore P\ lies\,inside\,the\,circle$.

48. Using the letters of the word TRICK, a five letter word witth distinct letters is formed such that C is in the middle. Tn how many ways this is possible?

1) 6 2) 120 3) 24 4) 72

Key: 3

Sol:

		C		

No of ways = 4! = 24.

49. The angle between the curves $x^2=8y$ and $xy=8$ is

1) $\tan^{-1}\left(\dfrac{-1}{3}\right)$ 2) $\tan^{1}\left(-3\right)$ 3) $\tan^{-1}\left(-\sqrt{3}\right)$ 4) $\tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$

Key: 2

Sol: $x^2=8y\ -\left(1\right)\quad xy=8\ -\left(2\right)$

Solving (1) and (2) we get $\left(x,y\right)=\left(4,2\right)$

$\tan\theta=\dfrac{m_1-m_2}{1+m_1m_2}$

$\therefore\theta=\tan^{-1}\left(-3\right)$.

50. $f:(-\infty,o]\to[0,\infty)$ **is defined as** $f(x)=x^2$**. The domain and range of its inverse is**

1) Domain of $\left(f^{-1}\right)=[0,\infty)$, Range of $\left(f^{-1}\right)=(-\infty,0]$

2) Domain of $\left(f^{-1}\right)=[0,\infty)$, Range of $\left(f^{-1}\right)=(-\infty,\infty)$

3) Domain of $\left(f^{-1}\right)=[0,\infty)$, Range of $\left(f^{-1}\right)=[0,\infty)$

4) f^{-1} doesnot exist

Key: 1

Sol: . $f(x)=x^2$

$f^{-1}(x)=y\Rightarrow x=f(x)$

$\Rightarrow x=y^2\Rightarrow y=\pm\sqrt{x}$

$f^{-1}(x)=\pm\sqrt{x}$

$\text{Domain}=[0,\infty)$

$\text{Range}=(-\infty,0]$

51. If $\vec{a},\vec{b}$ **and** $\vec{c}$ **are unit vectors such that** $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ **and** $\left(\vec{a},\vec{b}\right)=\frac{\pi}{3}$**, then** $\left|\vec{a}\times\vec{b}\right|+\left|\vec{b}\times\vec{c}\right|+\left|\vec{c}\times\vec{a}\right|=$

1) $\frac{3}{2}$

2) 0

3) $\frac{3\sqrt{3}}{2}$

4) 3

Key: 3

Sol: $\left|\bar{a}\right|=\left|\bar{b}\right|=\left|\bar{c}\right|=1$

$\bar{a}+\bar{b}+\bar{c}=0$

$\bar{b}\times\bar{a}\times\bar{b}\times\bar{c}=0$

$\bar{b}\times\bar{c}=\bar{a}\times\bar{b},\bar{c}\times\bar{a}=\bar{a}\times\bar{b}$

$\left|\bar{c}\times\bar{b}\right|+\left|\bar{b}\times\bar{c}\right|+\left|\bar{c}\times\bar{a}\right|=3\left|\bar{a}\times\bar{b}\right|$

$=3\left|\bar{a}\right|\left|\bar{b}\right|\sin\frac{\pi}{3}$

$=3.1.1.\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{2}$

52. The differential equation of the simple harmonic motion given by $x=A\cos\left(nt+\alpha\right)$ **is**

1) $\frac{d^2x}{dt^2}-n^2x=0$

2) $\frac{d^2x}{dt^2}+n^2x=0$

3) $\frac{dx}{dt}-\frac{d^2x}{dt^2}=0$

4) $\frac{d^2x}{dt^2}-\frac{dx}{dt}+nx=0$

Key: 2

Sol: $x=A\cos\left(nt+\alpha\right)$

$\frac{dn}{dt}=-A\sin\left(nt+\alpha\right)n$

$$\frac{d^2n}{dt^2}=-n^2A\cos\left(nt+\alpha\right)$$

$$\frac{d^2x}{dt^2}+n^2x=0$$

53. If $\vec{a}$ and $\vec{b}$ are unit vectors and α is the angle between them, $\vec{a}+\vec{b}$ is a unit bvector when $\cos\alpha=$

1) $-\frac{1}{2}$ 2) $\frac{1}{2}$ 3) $-\frac{\sqrt{3}}{2}$ 4) $\frac{\sqrt{3}}{2}$

Key: 1

Sol: $\left|\overline{a}\right|=\left|\overline{b}\right|=1\quad\left|\overline{a}\right|\left|\overline{b}\right|=1$

$$\left|a\right|^2+\left|\overline{b}\right|^2+2\overline{a}.\overline{b}=1$$

$$1+1+2\left|\overline{a}\right|\left|\overline{b}\right|\alpha=1$$

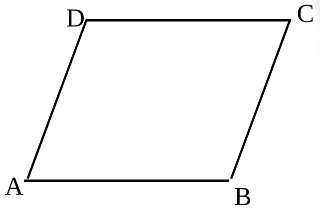
$$or\alpha=\frac{1}{2}.$$

54. A parallelogram has vertices $A(4,4,-1),B(5,6,-1),C(6,5,1)$ and $D(x,y,z)$. Then the vertex D is

1) $(5,1,0)$ 2) $(-5,0,1)$ 3) $(5,3,1)$ 4) $(5,1,3)$

Key: 3

Sol:



$$D=A+C-B$$
$$=(4+6-5,4+5-6,-1+1+1)$$
$$=(5,3,+1).$$

55. If $2x^2-10xy+2\lambda y^2+5x-16y-3=0$ represents a pair of straight lines, then point of intersection of theose lines is

1) $(2,-3)$ 2) $(5,-16)$ 3) $\left(-10,\frac{-7}{2}\right)$ 4) $\left(-10,,\frac{-3}{2}\right)$

Key: 3

Sol: $\frac{df}{dx}=0,\ \frac{df}{dy}=0$

$$hx-10y+5=0,\quad -10y+2xy-16=0$$

$$(x,y)=\left(-10,\frac{-7}{2}\right).$$

56. If rank of $\begin{pmatrix} x & x & x \\ x & x^2 & x \\ x & x & x+1 \end{pmatrix}$ **is 1, then**

1) $x=0(or)x=1$ 2) $x=1$ 3) $x=0$ 4) $x \neq 0$

Key: 3

Sol: $\begin{bmatrix} x & x & x \\ x & x^2 & x \\ x & x & x+1 \end{bmatrix}$

$R_2 \to R_2 - R_1,\ \ R_3 \to R_3 - R_1$

$$\begin{bmatrix} x & x & x \\ 0 & x(x-1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x=0\cdot$

57. If the vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+2\hat{k}$ **and** $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$ **are coplanar, then x=**

1) 1 2) 2 3) 0 4) -2

Key: 4

Sol: $\left[\overline{a}\overline{b}\overline{c}\right]=0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix}=0$$

$x(3)-(x-2)(1)-1(-2)$

$3x-x+2+2=0$

$2x+4$

$x=-2\cdot$

58. In order to eliminate the first degree terms from the equation $4x^2+8xy+10y^2-8x-44y+14=0$ **the point to which the origin has to be shifted is**

1) $(-2,3)$ 2) $(2,-3)$ 3) $(1,-3)$ 4) $(-1,3)$

Key: 1

Sol: $4x^2+8xy+10y^2-8x-4xy-14=0$

$\frac{dy}{dx}=01 \quad \frac{df}{dy}=0$

$8x+8y-8=0 \quad 8x+20y-44=0$

$x+y-1=0 \qquad 2x+5y-11=0$

$so1\&2(-2,3)=(x,y).$

59. Two circles of equal radius 'a' cut orthogonally. If their centres are (2,3) and (5,6), then radic al axis of these circles passes through the point

1) $(3a,5a)$ 2) $(2a,a)$ 3) $\left(a,\frac{5a}{3}\right)$ 4) (a,a)

Key: 3

Sol: $r_1=r_2=a \quad c_1=(2,3) \quad c_2=(5,6)$

$$d=c_1c_2=\sqrt{9+9}=\sqrt{18}$$

$d^2=r_1^2+r_2^2$

$18=a^2+a^2$

$2a^2=18 \ a^2=9$

$a=3$

$\left(\frac{7}{2},\frac{9}{2}\right)$

$y-\frac{9}{2}=-1\left(x-\frac{7}{2}\right)$

$2y-9=-2x+7$

$2x+2y-16=0$

$x+y-8=0$

By verification $x,y=\left(a,\frac{5a}{3}\right)$.

60. If $\tan\theta_1=k\cot\theta_2$, **then** $\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)}=$

1) $\frac{1+k}{1-k}$ 2) $\frac{1-k}{1+k}$ 3) $\frac{k+1}{k-1}$ 4) $\frac{k-1}{k+1}$

Key: 2

Sol: $\frac{\sin\theta_1}{\cos\theta_1}=k\frac{\cos\theta_2}{\sin\theta_2}$

$\frac{1}{k}=\frac{\cos\theta_1,\cos\theta_2}{\sin\theta,\sin\theta_2}$

$\frac{1-k}{1+k}=\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)}$.

61. Let $\vec{a}=2\vec{i}+\vec{j}-3\vec{k}$ **and** $\vec{b}=\vec{i}+3\vec{j}+2\vec{k}$**. Then the volume of the parallelopiped having coterminous edges as** $\vec{a}$**,** $\vec{b}$ **and** $\vec{c}$**, where** $\vec{c}$ **is the vector perpendicular to the plane of** $\vec{a}$**,** $\vec{b}$ **and** $|\vec{c}|=2$ **is**

1) $2\sqrt{195}$ 2) 4 3) $\sqrt{200}$ 4) $\sqrt{195}$

Key: 1

Sol: . $\vec{a}=2\bar{i}+\bar{j}-3\bar{k}$, $\bar{b}=\bar{i}+3\bar{j}+2\bar{k}$

$1\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\bar{a}\times\bar{b}.\bar{c}$

$= |\bar{a}\times\bar{b}||\bar{c}|\cos\left(\bar{a}\times\hat{b},\bar{c}\right)$

$$|\bar{a}\times\bar{b}|=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 2 & 1 & -3\\ 1 & 3 & 2\end{vmatrix}=\bar{i}(11)-\bar{j}(7)+\bar{k}(5)$$

$= 11\bar{i}-7\bar{j}+5\bar{k}$

$\left|\bar{a}\times\bar{b}\right|=\sqrt{121+49+25}=\sqrt{195}$

$\left|\bar{a}\quad\bar{b}\quad\bar{c}\right|=\left|\sqrt{195}.2.(\pm1)\right|=2\sqrt{195}$

62. The local maximum of $y=x^3-3x^2+5$ is attained at

1) x = 0　　2) x = 2　　3) x = 1　　4) x = - 1

Key: 1

Sol: $y=x^3-3x^2+5$

$\frac{dy}{dx}=3x^2-6x$, $\frac{dy}{dx}=0$ $3x^2-6x=0$

$\frac{d^2y}{dx^2}=6x-6$ $3x(x-2)=0$

$x=0,\ x=2$

$\left(\frac{d^2y}{dx^2}\right)_{x=0}=-6<0$, $\left(\frac{d^2y}{dx^2}\right)_{x=2}=12-6=6>0$

Local maximum　　Local minimum

63. In the expansion of $(1+x)^n$, the coefficients of p^{th} and $(p+1)^{th}$ terms are respectively p and q, then p + q =

1) n + 3　　2) n + 2　　3) n　　4) n + 1

Key: 4

Sol: $(1+x)^n$

$T_P=T_{(p-1)+1}={}^nC_{p-1}\,x^{p-1}\quad {}^nC_{p-1}=p$

$T_{P+1}={}^nC_p\,x^p\quad {}^nC_p=q$

$p+q={}^nC_{p-1}+{}^nC_p={}^{n+1}C_p$

$$\frac{p}{q}=\frac{{}^nC_{p-1}}{{}^nC_p}$$

$$\frac{p}{q}=\frac{p}{n-p+1}$$

$q = n - p + 1$

$p + q = n + 1$

64. **If** $f(x) = \begin{cases} \sin x & if\ x \le 0 \\ x^2 + a^2 & if\ 0 < x < 1 \\ bx + 2 & if\ 1 \le x \le 2 \\ 0 & if\ x > 2 \end{cases}$ **is continuous on R, then a + b + ab =**

1) - 2 2) 0 3) 2 4) - 1

Key: 4

Sol: $\underset{x \to 0^-}{Lt} f(x) = \underset{x \to 0^+}{Lt} f(x)$

$\underset{x \to 0}{Lt} \sin x = \underset{x \to 0}{Lt} x^2 + a^2$

$0 = a^2 \Rightarrow \boxed{a = 0}$

$\underset{x \to 2^-}{Lt} f(x) = \underset{x \to 2^-}{Lt} f(x)$

$\underset{x \to 2}{Lt} bx + 2 = \underset{x \to 2}{Lt} (0)$

$2b + 2 = 0$

$\boxed{b = -1}$

$a + b + ab = 0 - 1 + 0 = -1$

65. **If** $\cosh^{-1} x = 2\log_e\left(\sqrt{2} + 1\right)$**, then x =**

1) 1 2) 2 3) 4 4) 3

Key: 4

Sol: $\cosh^{=1} x = \log_e\left(\sqrt{2} + 1\right)^2$

$\log_e\left(x + \sqrt{x^2 - 1}\right) = \log_e\left(3 + \sqrt{9 - 1}\right)$

$\boxed{x = 3}$

66. **For any integer** $n \ge 1$**,** $\sum_{K=1}^{n} K(K+2)$ **=**

1) $\frac{n(n+1)(n+2)}{6}$ 2) $\frac{n(n+1)(2n+7)}{6}$ 3) $\frac{n(n+1)(2n+1)}{6}$ 4) $\frac{n(n-1)(2n+8)}{6}$

Key: 2

Sol: $\sum_{K=1}^{n} K(K+2)$

= $1.3 + 2.4 + 3.5 + 4.6 + ...$

= $\frac{n(n+1)(2n+7)}{6}$.

67. The foci of the ellipse $25x^2+4y^2+100x-4y+100=0$ **are**

1) $\left(\frac{5\pm\sqrt{21}}{10},-2\right)$ 2) $\left(-2,\frac{5\pm\sqrt{21}}{10}\right)$ 3) $\left(\frac{5\pm\sqrt{21}}{10},-2\right)$ 4) $\left(-2,\frac{2\pm\sqrt{21}}{10}\right)$

Key: 2

Sol: $25x^2+4y^2+100x-4y+100=0$

$$25(x^2+4x+4)+4\left(y^2-y+\frac{1}{4}\right)=-\cancel{100}+\cancel{100}+1$$

$$25(x+2)^2+4\left(y-\frac{1}{2}\right)=1$$

$$\frac{(x+2)^2}{\frac{1}{25}}+\frac{\left(y-\frac{1}{2}\right)^2}{\frac{1}{4}}=1 \qquad (h,k)\neq\left(-2,\frac{1}{2}\right)$$

$$a=\frac{1}{5},\ b=\frac{1}{2} \qquad \boxed{a<b}$$

Foci = $(h,k+be)$, $\quad e=\sqrt{\dfrac{\frac{1}{4}-\frac{1}{25}}{\frac{1}{4}}}=\dfrac{\sqrt{21}}{5}$

$$=\left(-2,\frac{1}{2}\pm\frac{1}{2},\frac{\sqrt{21}}{5}\right)$$

$$=\left(-2,\frac{5\pm\sqrt{21}}{10}\right)$$

68. $\left[\dfrac{1+\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)}{1+\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)}\right]^{72}=$

1) 0 2) - 1 3) 1 4) $\frac{1}{2}$

Key: 3

Sol: $\left[\dfrac{1+\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)}{1+\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)}\right]^{72}$

$$= \left[\frac{2\cos^2\frac{\pi}{24}+i\,2\sin\frac{\pi}{24}\cos\frac{\pi}{24}}{2\cos^2\frac{\pi}{24}-i\,2\sin\frac{\pi}{24}\cos\frac{\pi}{24}}\right]^{72}$$

$$= \frac{cis\,3\pi}{cis\left(-3\pi\right)}=cis\,6\pi=1$$

69. If the range of the function $f(x)=-3x-3$ is $\{3,-6,-9,-18\}$, then which of the following eements is not in the domain of f ?

1) - 1 2) - 2 3) 1 4) 2

Key: 1

Sol: By verification '-1' is not in the domain.

70. In ΔABC, if a = 1, b = 2, $\underline{|C}=60^{o}$ then $4\Delta^2+c^2=$

1) 6 2) 3 3) $\frac{\sqrt{3}}{2}$ 4) 9

Key: 1

Sol: $a=1, b=2, \underline{|C=60}$

$$4b^2+c^2=4\left(\frac{1}{2}ab\sin c\right)^2+a^2+b^2-2ab\cos c$$

$$= \cancel{4}.\frac{1}{\cancel{4}}1.\cancel{4}.\frac{3}{\cancel{4}}+1+4-2.1.\cancel{2}.\frac{1}{\cancel{2}}$$

=3 + 3 = 6

71. If the magnitude of $\vec{a}$, $\vec{b}$ and $\vec{a}+\vec{b}$ are respectively 3, 4 and 5, then the magnitude of $\left(\vec{a}-\vec{b}\right)$ is

1) 3 2) 4 3) 6 4) 5

Key: 4

Sol: $|\vec{a}|=3, |\vec{b}|=4, |\vec{a}+\vec{b}|=5$

$$|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2\left(|\vec{a}|^2+|\vec{b}|^2\right)$$

$$25+|\vec{a}-\vec{b}|^2=2\left(9+16\right)$$

$$= |\vec{a}-\vec{b}|^2=25$$

$$= |\vec{a}-\vec{b}|=5$$

72. If $\int f(x)\cos x\,dx=\frac{1}{2}\left(f(x)\right)^2+C$ and $f(0)=0$, then $f'(0)=$

1) 1 2) - 1 3) 0 4) 2

Key: 1

Sol: $f(x)=\sin x \Rightarrow f(0)=0$

$f'(x)=\cos x$

$f'(0)=\cos 0=1$

73. If α and β are the roots of the equation $ax^2+bx+c=0$ and the equation having roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$ is $px^2+qx+r=0$, then r =

1) a + 2b 2) ab + bc + ca 3) a + b + c 4) abc

Key: 3

Sol: $ax^2+bx+c=0$, $\alpha+\beta=\frac{-b}{a}$, $\alpha\beta=\frac{c}{a}$

Put $x=\frac{1-\alpha}{\alpha} \Rightarrow \alpha x=1-\alpha$

sub in (1) $\alpha(1+x)=1 \Rightarrow \alpha=\frac{1}{1+x}$

$$\frac{a}{(1+x)^2}+\frac{b}{1+x}+c=0 \Rightarrow a+b(1+x)+c(1+x)^2=0$$

$$cx^2+(b+2c)x+(a+b+c)=0$$

$\therefore$ r = a + b + c

74. If $A\left(\frac{\pi}{3}\right)$, $B\left(\frac{\pi}{6}\right)$ are the points on the circle represented in parametric form with centre (0, 0) and radius 12 then the length of the chord AB is

1) $6\left(\sqrt{6}-\sqrt{2}\right)$ 2) $6\left(\sqrt{6}-\sqrt{3}\right)$ 3) $\sqrt{2}\left(\sqrt{3}-1\right)$ 4) $6\left(\sqrt{3}-1\right)$

Key: 1

Sol: Length of chord AB = $2a\sin\left(\frac{A-B}{2}\right)$

$= 2(12)\sin(15)$

$= \cancel{2}\times\overset{6\sqrt{2}}{\cancel{12}}.\frac{\sqrt{3}-1}{\cancel{2}\cancel{\sqrt{2}}}=6\left(\sqrt{6}-\sqrt{2}\right)$

75. If the pair of straight lines $xy-x-y+1=0$ and the line $x+ay-3=0$ are concurrent, then the acute angle between the pair of lines $ax^2-13xy-7y^2+x+23y-6=0$ is

1) $\cos^{-1}\left(\frac{5}{\sqrt{218}}\right)$ 2) $\cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$ 3) $\cos^{-1}\left(\frac{5}{\sqrt{173}}\right)$ 4) $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Key: 2

Sol: $xy-x-y+1=0\,,\qquad x+ay-3=0$

$(x-1)(y-1)=0\qquad 1+a-3=0\qquad\qquad 2x^2-13xy-7y^2+x+23y-6=0$

$x-1=0$

$y-1=0\qquad\qquad \boxed{a=2}\qquad\qquad \cos\theta=\frac{15}{\sqrt{81+169}}=\frac{\cancel{5}}{\cancel{5}\sqrt{10}}$

$(x,y)=(1,1)\qquad\qquad \theta=\cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$

76. The number of solutions of $\cos 2\theta=\sin\theta$ in $(0,2\pi)$ is

1) 4 2) 3 3) 2 4) 5

Key: 2

Sol: $\cos 2\theta=\sin\theta$

$1-2\sin^2\theta=\sin\theta$

$2\sin^2\theta+\sin\theta-1=0$

$\left(2\sin\theta-1\right)\left(\sin\theta+1\right)=0$

$\sin\theta=\frac{1}{2}\qquad\qquad \sin\theta=-1$

$\theta=n\pi+(-1)^n\frac{\pi}{6}\qquad\qquad \theta=n\pi+(-1)^n\left(-\frac{\pi}{2}\right)$

$n=0\ \Rightarrow\ \theta=\frac{\pi}{6}$

$n=1\ \Rightarrow\ \theta=\frac{5\pi}{6}$

$n=2\Rightarrow\theta=2\pi-\frac{\pi}{2}=3\frac{\pi}{2}$

number of solutions = 3

77. The lengths of the sides of a triangle are 13, 14 and 15. If R and r respectively denote the circum radius and inradius of that triangle, then 8R + r =

1) 84 2) $\frac{65}{8}$ 3) 4 4) 69

Key: 4

Sol: a = 13, b = 14, c = 15

$s=\frac{a+b+c}{2}=21$

$\Delta=\sqrt{21.8.7.6}=84$

$R=\frac{abc}{4\Delta}=\frac{13+4.\cancel{15}^{5}}{4(8)\cancel{6}_{2}}=\frac{65}{8}$

$r=\frac{\Delta}{5}=\frac{84}{21}=4$

$8R+r=65+4=69$

78. If A and B are the variances of the 1st 'n' even numbers and 1st 'n' odd numbers respectively then

1) A = B 2) A > B 3) A < B 4) A = B + 1

Key: 1

Sol: By simplying A = B

79. If the line $x-y=-4K$ is a tangent to the parabola $y^2=8x$ at P, then the perpendicular distance of normal at P from (K, 2K) is

1) $\frac{5}{2\sqrt{2}}$ 2) $\frac{7}{2\sqrt{2}}$ 3) $\frac{9}{2\sqrt{2}}$ 4) $\frac{1}{2\sqrt{2}}$

Key: 3

Sol: $y^2=8x$ .... (1) $\Rightarrow$ a = 2

$y=x+4K$ .... (2) as m = 1

$c=\frac{a}{m}$ $\Rightarrow$ $4K=2$

$$\boxed{K=\frac{1}{2}}$$

$P\left(\frac{1}{2},1\right)$

from of normal $y=mx-xm-am$

$y=-x+4+2$

$x+y-6=0$

Perpendicular = $\frac{\left|\frac{1}{2}+1-6\right|}{\sqrt{2}}=\frac{9}{2\sqrt{2}}$

80. If A and B are evens having probabilities, P(A) = 0.6, P(B) = 0.4 and $P(A\cap B)=0$, then the probability that neither A nor B occurs is

1) $\frac{1}{4}$ 2) 1 3) $\frac{1}{2}$ 4) 0

Key: 4

Sol: $P\left|\bar{A}\cap\bar{B}\right|=P\left(\overline{A\cup B}\right)$

$=1-P\left(A\cup B\right)$

$=1-\left[0.6+0.4-0\right]$

$=0$

PHYSICS

81. A force $\overline{F}$ is applied on a square plate of length L. If the percentage error in the determination of L is 3% and in F is 4% , the permissible error in the calculation of pressure is

1) 13% 2) 10% 3) 7 % 4) 12 %

Key: 2

Sol: $\frac{\Delta p}{P}=\frac{\Delta F}{F}+2\frac{\Delta L}{L}$

$=4\%+2\times3\%$

$=10\%$

82. A positive charge ‘Q’ is placed on a conductiing spherical shell with inner radius R_1 and outer radius R_2. A particle with charge ‘q’ is placed at the center of the spherical cavity. The magnitude of the electric field at a point in the cavity, a distance ‘r’ from center is

1) zero 2) $\frac{Q}{4\pi\,\varepsilon_0\,R^2}$ 3) $\frac{q}{4\pi\,\varepsilon_0\,R^2}$ 4) $\frac{(q+Q)}{4\pi\,\varepsilon_0\,R^2}$

Key: 3

Sol: $E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

83. A swimmer wants to cross a 200 m wide river which is flowing at a speed of 2 m/s. The velocity of the swimmer with respect to the river is 1 m/s. How far from the point directly opposite to the starting point does the seimmer reach the opposite bank?

1) 200 m 2) 400 m 3) 600 m 4) 800 m

Key: 2

Sol:

84. A coil having ‘n’ turns and resistance $R\Omega$ is connected with a galvanometer of resistance $4R\Omega$ This combination is moved in time ‘t’ seconds from a magnetic flux ϕ_1 weber to ϕ_2 weber. The induced current in the circuit is

1) $\frac{\phi_2-\phi_1}{5Rnt}$ 2) $\frac{n(\phi_2-\phi_1)}{5Rnt}$ 3) $-\frac{(\phi_2-\phi_1)}{Rnt}$ 4) $-\frac{n(\phi_2-\phi_1)}{5Rt}$

Key: 2

Sol: $\tau=\frac{-n(\phi_2-\phi_1)}{(4R+R)t}$

85. A Simple pendulum of length 1m is freely suspended from the ceiling of an elevator. The time period of small oscillations as the elevator moves up with an acceleration of $2m/s^2$ is (use $g=10m/s^2$)

1) $\frac{\pi}{\sqrt{5}}s$ 2) $\sqrt{\frac{2}{5}}\pi s$ 3) $\frac{\pi}{\sqrt{2}}s$ 4) $\frac{\pi}{\sqrt{3}}s$

Key: 4

Sol: $F=2\pi\sqrt{\frac{l}{g+a}}$

$$= 2\pi\sqrt{\frac{1}{12}}$$

$$= \pi\sqrt{\frac{4}{12}} = \frac{\pi}{\sqrt{3}}$$

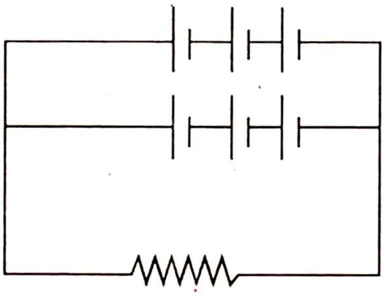
86. Consider a metal ball of radius ‘r’ moving at a constant velocity ‘v’ in a uniform magnetic field of induction $\overline{B}$. Assuming that the direction of velocity forms an angle ‘α’ with the dirction of $\overline{B}$, the maximum potential difference between points on the ball is

1) $r\left|\overline{B}\right|\left|\overline{v}\right|Sin\alpha$ 2) $\left|\overline{B}\right|\left|\overline{v}\right|Sin\alpha$ 3) $2r\left|\overline{B}\right|\left|\overline{v}\right|Sin\alpha$ 4) $2r\left|\overline{B}\right|\left|\overline{v}\right|Cos\alpha$

Key: 1

Sol: $e = r\ B.V\sin\alpha$

87. Each of the six ideal batteries of emf 20V is connected to an external resistance of 4Ω as shown in the figure. The current through the resistance is



R = 4Ω

1) 6 A 2) 3 A 3) 4 A 4) 5 A

Key:

Sol: Conceptual

88. The energy that should be added to an electron to reduce its de-Broglie wavelenght from 1 nm to 0.5 nm is

1) four - time the initial energy 2) equal to the inital energy

3) two - times the initial energy 4) three - times the inital energy

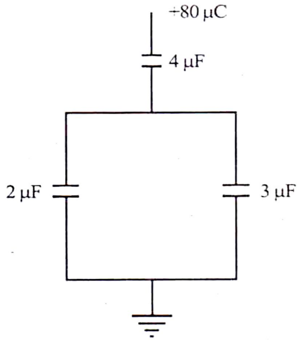
Key: 4

Sol: $E = \frac{h^2}{2m\lambda^2}\ E\alpha\frac{1}{\lambda^2}$

$$\frac{E_2}{E_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^2 = \left(\frac{0.5}{1}\right)^2$$

$$E_2 = 4E_1$$

89. In the given circuit, a charge of $+80\mu C$ is given to upper plate of a $4\mu F$ capacitor. At steady state the charge on the upper plate of the $3\mu F$ capacitor is :



1) 60 μC 2) 48 μC 3) 80 μC 4) 0 μC

Key: 2

Sol: $V = \frac{Q_e}{C_e} = \frac{Q_1}{C_1} = \frac{80}{5} = 16$

$Q_1 = C_1 V_1 = 16 \times 3 = 48\mu C$

90. The Young's modulus of a material is $2\text{x}10^{11} N/m^2$ and its elastic limit is $1\text{x}10^8 N/m^2$. For a wire of 1 m length of this material, the maximum elongation achievable is

1) 0.2 mm 2) 0.3 mm 3) 0.4 mm 4) 0.5 mm

Key: 4

Sol: $Y = \frac{F}{A} . \frac{l}{e}$

$e = \frac{F}{A} . \frac{l}{y} = \frac{1 \times 10^8}{2 \times 10^4}$

$= 0.5 \times 10^{-3} = 0.5mm$

91. A wooden box lying at rest on an inclined surface of a wet wood is held at static equilibrium by a constant force $\vec{F}$ applied perpendicular to the incline. If the mass of the box is 1 kg, the angle of inclination is 30^0 and the coeffcient of static friction between the box and the inclined plane is 0.2, the minimum magnitude of $\vec{F}$ is (Use g = 10 m/s^2)

1) 0 N, as 30^0 is less than angle of repose 2) $\geq 1N$

3) $\geq 3.3N$ 4) $\geq 16.3N$

Key: 3

Sol: $F_{\min} = \frac{mg}{\mu}(\sin\theta - \mu\cos\theta)$

$= \frac{1 \times 10}{0.2}\left(\frac{1}{2} - 0.2 \times \frac{\sqrt{3}}{2}\right)$

$= 50(0.5 - 0.1732)$

$= 50 \times 0.3268$

$= 16.34N$

92. A metre scale made of steel reads accurately at 25^0C. Suppose in an experiment an accuracy of 0.06 mm in 1m is required, the range of temperature in which the experiment can be performed with this metre scale is (coefficient of linear expansion of steel is $11x10^{-6}/^0C$)

1) $19^0C\,to\,30^0C$ 2) $25^0C\,to\,32^0C$ 3) $18^0C\,to\,25^0C$ 4) $18^0C\,to\,32^0C$

Key: 2

Sol: $\Delta l = l \times \Delta t$

93. Consider a solenoid carrying current supplied by a DC source with a constant emf contaiing iron core inside it. When the core is pulled out of the solenoid, the change in current will

1) remain same 2) decrease 3) increase 4) modulate

Key: 1

Sol: Conceptual

94. A parallel beam of light of intensity I_0 is incident on a coated glass plate. If 25% of the incident light is reflected from the upper surface and 50% of light is reflected from the lower surface of the glass plate, the ratio of maximum to minimum intensity in the interference region of the reflected light is

1) $\left(\dfrac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ 2) $\left(\dfrac{\frac{1}{4}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ 3) $\frac{5}{8}$ 4) $\frac{8}{5}$

Key: 1

Sol: $\dfrac{I_{\max}}{I_{\min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$

$I_1 = 25\%\ I_0 = \frac{1}{4}I_0$

$I_2 = \frac{3}{4}\times\frac{1}{2}I_0$

95. A thermocol box has a total area (including the lid) of 1.0 m^2 and wall thickness of 3cm. It is filled with ice at $0\ ^0C$. If the average temperature outside the box is 30^0C throughout the day, the amount of ice that melts in one day is

1) 1 kg 2) 2.88 kg 3) 25.92 kg 4) 8.64 kg

Key: 4

Sol: $\dfrac{m\,vice}{t} = \dfrac{K_A\left(\theta_1-\theta_2\right)}{l}$

$$\frac{m\times3\times10^5}{86400} = \frac{3\times10^{-2}\times1}{3\times10^{-2}}$$

$m = 8.64\,kg$

96. Which of the following is emitted when ${}^{239}_{94}Pu$ decays into ${}^{235}_{92}U$?

1) Gamma Ray 2) Neutron 3) Electron 4) Alpha particle

Key: 4

Sol: ${}_{94}Pu^{239} \xrightarrow{Limits_{(\alpha)}} {}_{92}U^{235} + {}_{2}He^{4}$

97. An AC generator producing 10 V (rms) at 200 rad/s is connected in series with a 50Ω resistor, a 400mH inductor and a 200 μF capacitor. The rms voltage across the inductor is

1) 2.5 V 2) 3.4 V 3) 6.7 V 4) 10.8 V

Key:

Sol: $Z = \sqrt{\left(x_L - x_C\right)^2 + R^2}$

$I = 74.33\Omega$

$I = \frac{v}{z} = \frac{10}{74.33} v = I \times L = \frac{10}{74.33} \times 80$

98. Wire has resistance of 3.1Ω at $30\,{}^0C$ and 4.5Ω at $100^0\,C$. The temperature coefficient of resistance of the wire is

1) 0.0012 ${}^0C^{-1}$ 2) 0.0024 ${}^0C^{-1}$ 3) 0.0032 ${}^0C^{-1}$ 4) 0.0064 ${}^0C^{-1}$

Key:

Sol: $\alpha = \frac{R_2 - R_1}{R_1\left(t_2 - t_1\right)}$

$\frac{4.5 - 3.1}{3.1(100 - 30)} = \frac{1.4}{3.1 \times 70} = 0.0064$

99. An object is thrown vertically upward with a speed of 30 m/s. The velocity of theobject half-a-second before it reaches the maximum height id

1) 4.9 m/s 2) 9.8 m/s 3) 19.6 m/s 4) 25.1 m/s

Key: 1

Sol: .$U = gt \; t = \frac{4}{g} = \frac{30}{9.8} = 3.06$ $t^1 = 3.06 - 0.5 - = 2.56$

$V = u - gt^1 = 30 - (2.56) = 4.9\,m/\sec$

100. An electron collides with a Hydrogen atom in its ground state and excites it to n = 3 state. Theenergy given to the Hydrogen atom in this inelastic collision (neglecting the recoil of Hydrogen atom) is

1) 10.2 eV 2) 12.1 eV 3) 12.5 eV 4) 13.6 eV

Key: 2

Sol: .$\Delta E = E_2 - E_1 = 13.6 - 1.5 = 12.1ev$

101. Consider the mition of a particle described by $x = a\cos t, y = a\sin t\, and\; z = t.$ The trajectory traced by the particle as a function of time is

1) Helix 2) Circular 3) Elliptical 4) Straight line

Key: 2

Sol: $\sqrt{x^2 + y^2 + z^2} = \sqrt{a^2 \sin^2 t + a^2 \cos^2 t + t^2}$

102. Consider a reversible engine of efficiency 1/6, when the temperaure of the sink is reduced by 62^0C, its efficiency gets doubled. The temperature of the source and sink respectively are

1) 372 K and 310 K 2) 273 K and 300 K 3) $99^0 C\, and\, 10^0 C$ 4) $200^0 C\, and\, 37^0 C$

Key: 1

Sol: $n = \frac{T_2}{T_1} - 1$

$$\frac{T_2}{T_1} = 1 + \frac{1}{6} = \frac{7}{6}$$

$$T_2 = \frac{7}{6} T_1$$

$$\frac{1}{3} = \frac{T_2 - 62}{T_1} - T_2 = T = 62$$

$$T_1 = 372k$$

$$T_2 = 310k$$

103. Consider a light source placed at a distance of 1.5 m along the axis facing the convex side of a spherical mirror of raius of curvature 1 m. The position (s'), natureand magnification (m) of the image are

1) s'= 0.375 m, Virtual, upright, m = 0.25 2) s'= 0.375 m, Real, inverted, m = 0.25

3) s' = 3.75 m, Virtual, inverted, m = 2.5 4) s'=3.75 m, Real, upright, = 2.5

Key: 1

Sol: . $-\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{v} = -\left(\frac{1}{f} + \frac{1}{u}\right) = \frac{-8}{3}$$

$$v = \frac{3}{8} = 0.375m$$

Magnification $m = \frac{v}{u} = \frac{0.375}{1.5} = 0.25$

104. An office room contains about 2000 moles of air. The change in the internal energy of this much air when it is cooled from 34^0C to 24^0C at a constant pressure of 1.0 atm is [Use $\gamma_{air} = 1.4$ and Universal gas constant = 8.314 J/mol K]

1) $-1.9\,x 10^5 J$ 2) $+1.9\,x 10^5 J$ 3) $-4.2\,x 10^5 J$ 4) $+0.7\,x 10^5 J$

Key: 3

Sol: $\Delta Q = \Delta U + 2Rdt$

$$\Delta U = \Delta Q - nR\Delta t$$

$$\Delta U = \frac{nr^1 R}{r^1 - 1}\Delta t - nR\Delta t$$

$$= nR\Delta t\left(\frac{1}{r^1 - 1}\right)$$

$$= 4.2 \times 10^5 J$$

105. A ball is thrown at a speed of 20 m/s at an angle of 30^0 with the horizontal. The maximum height reached by te ball is (use $g = 10m/s^2$)

1) 2 m 2) 3 m 3) 4 m 4) 5 m

Key: 4

Sol: $H = \frac{u^2 \sin^2 \vartheta}{2g} = \frac{400}{2 \times 10 \times 4} = 5$

106. A horizontal pipeline carrying gasoline has a cross-sectional diameter or 5 mm. If the viscosity and density of the gasoline are $6 \text{x} 10^{-3}$ Poise and 720 kg/m^3 respectively, the velocity after which the flow becomes turbulent is

1) > 1.66 m/s 2) > 3.33 m/s 3) $> 1.6 \text{x} 10^{-3}\, m/s$ 4) > 0.33 m/s

Key: 3

Sol: $v_c = \frac{\eta}{dt} = \frac{6 \times 10^{-3}}{720 \times 5 \times 10^{-3}}$

$=1.66 \times 10^{-3}$

107. A piece of copper and a piece of germanium are cooled from room temperature to 80 K. Then, which one of the following is correct?

1) Resistance of each will increase 2) Resistance of each will decrease

3) Resistance of copper will decrease while that of germanium will increase

4) Resistance of copper will increase while that of germanium will decrease

Key: 3

Sol: $\begin{matrix} Cu \rightarrow Metal \\ Ge \rightarrow semconduct \end{matrix}$ Conceptual

108. A beam of light propagating at an angle α_1 from a medium 1 throgh to another medium 2 at an angle α_2. If the wavelength of light in medium 1 is λ_1, the wavelength of light in medium 2 (λ_2), is

1) $\frac{Sin\alpha_2}{Sin\alpha_1}\lambda_1$ 2) $\frac{Sin\alpha_1}{Sin\alpha_2}\lambda_1$ 3) $\left(\frac{\alpha_1}{\alpha_2}\right)\lambda_1$ 4) λ_1

Key: 1

Sol: $\frac{u_2}{u_1} = \frac{v_1}{v_2} - \frac{\lambda_1}{\lambda_2} = \frac{\sin\alpha_1}{\sin\alpha_2}$

$\lambda_2 = \frac{\sin\alpha_2}{\sin\alpha_1}.\lambda_1$

109. An amplitude modulated signal consists of a message signal of frequency 1 KHz and peak voltage of 5V, modulating a carrier frequency of 1 MHz and peak voltage of 15V. The correct description of this signal is

1) $5\left[1+3\sin\left(2\pi 10^6\right)t\right]\sin\left(2\pi 10^3 t\right)$ 2) $15\left[1+\frac{1}{3}\sin\left(2\pi 10^3\right)t\right]\sin\left(2\pi 10^6 t\right)$

3) $\left[5+15\sin\left(2\pi 10^3\right)t\right]\sin\left(2\pi 10^6 t\right)$ 4) $\left[15+5\sin\left(2\pi 10^6\right)t\right]\sin\left(2\pi 10^3 t\right)$

Key: 2

Sol: . $v_0\left[1+\frac{Am}{Ac}\sin w_m f\right]\sin w_c t$

$$15\left[1+\frac{1}{3}\sin\left(2\pi\times10^3(-)\right)\right]\sin\left(2\pi\times10^6(-)\right)$$

110. Which of the following principles is being used in sonar technology?

1) Newton's laws of motion
2) Reflection of electromagnetic waves
3) Laws of thermodynamics
4) Reflection of ultrasonic waves

Key:

Sol: Conceptual

111. A particle of mass M is moving in a horizontal circle of radius R with uniform speed v. When the particle moves from one point to a diametrically opposite point, its

1) momentum does not change
2) momentum changes by 2 Mv
3) kinetic energy changes by $\frac{Mv^2}{4}$
4) kinetic energy changes by Mv^2

Key:

Sol: $p_i - p_f = Mv-(-Mv) = 2Mv$

112. A billiard ball of mass 'M', moving with velocity 'v_1' collides with another ball of the same mass but at rest. If the collsion is elastic the angle of divergence after the collision is

1) 0^0
2) 30^0
3) 90^0
4) 45^0

Key:

Sol: .Conceptual. (Similar may when collides elastically exchange their celocities)

113. A planet of mass 'm' moves in an elliptical orbit around an unknown star of mass 'M' such that its maximum and minimum distances from the star are equal to r_1 and r_2 respectively. The angular moment of the planet relative to the centre of the star is

1) $m\sqrt{\frac{2GMr_1r_2}{r_1+r_2}}$
2) 0
3) $m\sqrt{\frac{2GM(r_1+r_2)}{r_1r_2}}$
4) $\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$

Key: 3

Sol: $ress = \frac{2(r_1+r_2)}{r_1r_2}$

$L = mvr$

$= m\sqrt{\frac{GM}{r}}r^2$

$= m\sqrt{\frac{2GM(r_1+r_2)}{r_1r_2}}$

114. Consider a frictionless ramp on which a mooth object is made to slide down from an initial height 'h'. The distance 'd' necessary to stop the object on a flat track (of coefficient of friction 'μ'), kept at the ramp end is

1) h/μ
2) μh
3) $\mu^2 h$
4) $h^2\mu$

Key: 1

Sol: $mgh = \mu mgs$

$$s = \frac{h}{\mu}$$

115. A generator with a circular coil of 100 turns of area $2\,\text{x}\,10^{-2}m^2$ is immersed in a 0.01 T magnetic field and rotated at a frequency of 50 Hz. The maximum emf which is produced during a cycle is

1) 6.28 V 2) 3.44 V 3) 10 V 4) 1.32 V

Key: 1

Sol: $\varepsilon = BNAw = 10^{-2}\,\text{x}\,10^{2}\,\text{x}\,2\,\text{x}10^{-2}\,\text{x}100^{\pi}$

= 2 x 3.14=6.28

116. A sound wave of frequency 'v' Hz initially travels a distance of 1 km in air. Then it getsreflected into a water reservoir of depth 600m. The frequency of the wave at the bottom of the reservoir is $\left(V_{air} = 340\,m/s; V_{water} = 1484m/s\right)$

1) > v Hz 2) < v Hz 3) v Hz

4) 0 (the sound wave gets attenuated by water completely)

Key: 3

Sol: .n = constant (conceptual)

117. Which of the following staetmentis not true?

1) the resistance of an itrinsic semiconductor decreases with increase in temperature

2) doping pure Si with trivalent impurities gives p-type semiconductor

3) the majority carriers in n-type semiconductors are holes

4) a p-n junction can act as a semiconductor diode

Key: 3

Sol: . conceptual

118. The decelration of a car traveling on a straight is a function of its instantaneous velocity 'v' given by $w = a\sqrt{v},$ where 'a' is a constant. If the initial velocity of the car is 60 km/hr, the distance the car will travel and the time it take before it stops are

1) $\frac{2}{3}m, \frac{1}{2}s$ 2) $\frac{3}{2a}m, \frac{1}{2a}s$ 3) $\frac{3a}{2}m, \frac{a}{2a}s$ 4) $\frac{2}{3a}m, \frac{2}{a}s$

Key: 4

Sol: $\frac{dv}{df} = a\sqrt{v}$

$$\frac{dv}{\sqrt{v}} = a.dt$$

$$\int_{v_f}^{v_i} \frac{dv}{\sqrt{v}} = a.dt$$

$$\left[2\sqrt{v}\right]_{v_f}^{v_i} = at$$

$$t = \frac{2}{a}v^{1/2}$$

$$-\frac{dv}{dt} = a\sqrt{v}$$

$$\frac{dv}{dx}.\frac{dx}{dt} = -a\sqrt{v}$$

$$\frac{dv}{dx}.v = -a\sqrt{v}$$

$$dv\sqrt{v} = -a.dx$$

$$\int_{v_0}^{0} \sqrt{v}\ dv = -a\int_{o}^{s} ds$$

$$= \frac{2}{3a} v_0^{3/2}$$

119. A current carrying wire in its neighbourhood produces

1) electric field
2) electric and magnetic fields
3) magnetic field
4) no field

Key: 3

Sol: Conceptual

120. Consider a particle on which constant forces $\overrightarrow{F_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ N and $\overrightarrow{F_2} = 4\hat{i} - 5\hat{j} - 2\hat{k}$ act together resulting in a displacement from postion $\vec{r_1} = 20\hat{i} + 15\hat{j}$ cm to $\vec{r_2} = 7\hat{k}$ cm. The total work done on the particle is

1) - 0.48 J
2) + 0.48 J
3) - 4.8 J
4) +4.8 J

Key:

Sol: $\left(\overline{F_1} + \overline{F_2}\right)\left(\overline{r_2} - \overline{r_1}\right)$

$$\left(5i - 3j + K\right)\left(-20i - 15j + 7K\right)\text{x}10^{-2}$$

$$= -48\,\text{x}\,10^{-2}$$

$$= -0.48$$

CHEMISTRY

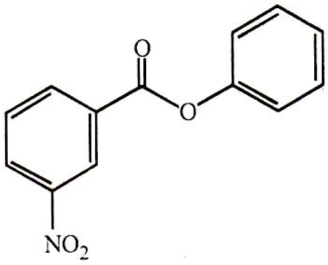
121. Which of the following conditions are correct for real solutions showing negative deviation from Raoult's law?

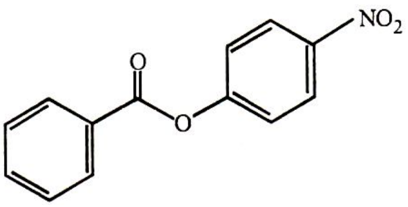
1) $\Delta G_{Mix} < 0; \Delta V_{Mix} > 0$
2) $\Delta H_{Mix} < 0; \Delta V_{Mix} > 0$
3) $\Delta H_{Mix} > 0; \Delta V_{Mix} < 0$
4) $\Delta H_{Mix} < 0; \Delta V_{Mix} < 0$

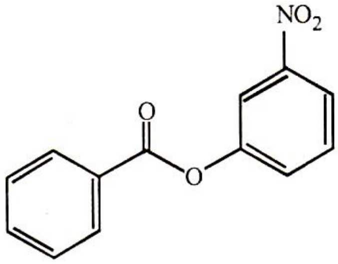
Key: 4

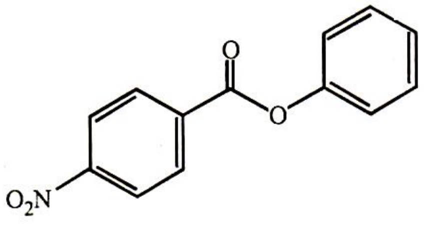
Sol: Conceptual

122. Nitration of phenyl benzoate yields the product

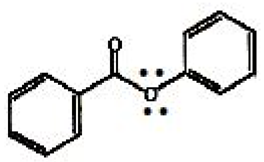
1) 

2) 

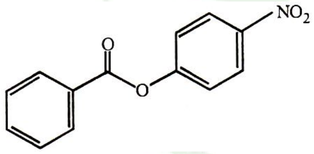
3) 

4) 

Key: 2

Sol: .  . Phenyl group connected to oxygen becomes activated for electrophilic substitution reaction due to the presence of lone pair electrons on oxygen (it is +M effect)

$\therefore$ Substitution of $-NO_2$ group takes place at para position of this phenyl group.

$\therefore$ The product is 

123. The electronic configuration of ${}_{59}\mathrm{Pr}$ (praseodimium) is

1) $\left[{}_{54}Xe\right]4f^{2}5d^{1}6s^{2}$ 2) $\left[{}_{54}Xe\right]4f^{1}5d^{2}6s^{2}$ 3) $\left[{}_{54}Xe\right]4f^{3}6s^{2}$ 4) $\left[{}_{54}Xe\right]4f^{3}5d^{2}$

Key: 1

Sol: Conceptual

124. Which of the following is the most basic oxide?

1) SO_3 2) SeO_3 3) PoO 4) TeO

Key: 3

Sol: Polonium is most metallic in the options given

$\therefore$ its oxide (PoO) is more basic

125. The element that forms stable compounds in low oxidation state is

1) Mg 2) Al 3) Ga 4) Tl

Key: 4

Sol: Thallium, due to inert pair effect shows stable lower oxidation state.

126. Atomic radius(pm) of Al, Si, N and F respectively is

1) 117, 143, 64, 74 2) 143, 117, 74, 64 3) 143, 47, 64, 74 4) 64, 74, 117, 143

Key: 2

Sol: Al, Si belong to 3rd period while O, F to second period. The size decreases from left to right in a period.

$\therefore$ Order of size is Al>Si> O> F.

127. Reaction of calgon with hard water containing Ca^{2+} ions produce

1) $\left[Na_2CaP_6O_{18}\right]^{2-}$ 2) $Ca_2\left(PO_4\right)_3$ 3) $CaCO_3$ 4) $CaSO_4$

Key: 1

Sol: $$\underset{\text{Calgon}}{\mathrm{Ca^{+2}+\left[Na_4P_6O_{18}\right]^{-2}}} \rightarrow \mathrm{\left[Na_2CaP_6O_{18}\right]^{-2}+2Na^{+}}$$

128. Which of the following statement is true

1) The pressure of a fixed amount of an ideal gas is proportional to its temperature only

2) Frequency of collisions increases in proportion to the square root of temperature

3) The value of van der Waal's constant 'a' is smaller for ammonia than for nitrogen

4) If a gas is expanded at constant temperature, the kinetic energy of the molecules decreases

Key: 1

Sol: As per PV= nRT equation, for a given amount of gas $P \propto T$.

129. Conversion of esters to aldehydes can be accomplished by

1) Stephen reduction 2) Rosenmund reduction

3) Reduction with lithium aluminium hydride 4) Reduction with diisobutyl aluminium hydride

Key: 4

Sol: Diisobutyl aluminium hydride(DIBAL) is a specific reagent to reduce esters to aldehydes.

130. Consider the following electrode processes of a cell,

$$Cl^- \rightarrow \frac{1}{2}Cl_2 + e^-$$

$$\left[MCl + e^- \rightarrow M + Cl^-\right]$$

If EMF of this cell is $-1.140V$ and E^0 value of the cell is $-0.55V$ at 298K, the value of the equilibrium constant of the sparingly soluble salt MCl is in the order of

10^{-10} **2)** 10^{-8} **3)** 10^{-7} **4)** 10^{-11}

Key: 1

Sol: $MCl + e^- \rightarrow M + Cl^-$ cathode (Reduction)

$Cl^- \rightarrow \frac{1}{2}Cl_2 + e^-$ Anode (Oxidation)

$$\boxed{MCl \rightarrow M + \frac{1}{2}Cl_2}$$

The K_C of the cell reaction is calculated from nernst equation

$$E_{Cell} = E^0_{cell} - \frac{0.059}{n}\log K_C$$

$$-1.140 = -0.55\frac{-0.059}{1}\log K_C$$

$$-0.59 = -0.059\log K_C$$

$\log K_C = \frac{0.59}{0.059} = 10$

$\therefore K_C = 10^{10}$

K_{SP} **is for** $M + \frac{1}{2}Cl_2 \to MCl \to M^+ + Cl^-$

$\therefore K_{Sp} = \frac{1}{K_C} = \frac{1}{10^{10}} = 10^{-10}$

131. Which of the following is true for spontaneous adsorption of H_2 gas without dissociation on solid surface.

1) Process is exothermic and $\Delta S < 0$
2) Process is endothermic and $\Delta S > 0$
3) Process is exothermic and $\Delta S > 0$
4) Process is endothermic and $\Delta S < 0$

Key: 1

Sol: Adsorption of H_2 gas on solid surface is exothermic and entropy decreases in the process.

$\therefore \Delta S < 0$

132. Consider the single electrode process $4H^+ + 4e^- \rightleftharpoons 2H_2$ catalyzed by platinum black electrode in HCl electrolyte. The potential of the electrode is $-0.059V$ Vs.SHE. What is the concentration of the acid in the hydrogen half cell if the H_2 pressure is 1 bar?

1) 1 M
2) 10 M
3) 0.1 M
4) 0.01 M

Key: 3

Sol: $E_{R.P} = 0.059\log\left[H^+\right]$

$-0.059 = 0.059\log\left[H^+\right]$

$\log\left[H^+\right] = -1$

$\left[H^+\right] = 10^{-1}M$

133. Which of the following elements has the lowest melting point?

1) Sn
2) Pb
3) Si
4) Ge

Key: 1

Sol: The melting point order of IVA group elements is

$C > Si > Ge > Pb > Sn$

134. The number of complementary hydrogen bond(s) between a guanine and cytosine pair is

1) 2
2) 1
3) 4
4) 3

Key: 4

Sol: The number of hydrogen bonds between adenine and thymine are two(A=T) Where as the number of hydrogen bonds between Guanine and cytosine are three

$(G \equiv C)$

135. Given ΔH_{f^0} for $CO_{2(g)}, CO_{(g)}$ and $H_2O_{(g)}$ are $-393.5, -110.5$ and $-241.8\ kJ\ mol^{-1}$, respectively. The $\Delta H_{f^0}\left[in\ kJ\ mol^{-1}\right]$ for the reaction $CO_{2(g)} + H_{2(g)} \to CO_{(g)} + H_2O_{(g)}$ is

1) 524.1
2) - 262.5
3) - 41.7
4) 41.2

Key: 4

Sol: $\Delta H_r = \left[{}^{\Delta H} f_{CO} + {}^{\Delta H} f_{H_2O}\right] - \left[{}^{\Delta H} f_{CO_2}\right]$

$= -110.5 - 241.8 + 393.5 = 41.2$

136. Which among the following is the strongest acid?

1) HF	2) HCl	3) HBr	4) HI

Key: 4

Sol: Among hydrogen halides the strongest acid is HI.

As the size of halogen increase, bond length increases hence bond energy decreases.

$\therefore$ HI releases $H^{\oplus}$ ions easily.

137. The species having pyramidal shape according to VSEPR theory is

1) SO_3	2) BrF_3	3) SiO_3^{2-}	4) OsF_2

Key: NO KEY

Sol: OsF_2 should be given as OSF_2 .

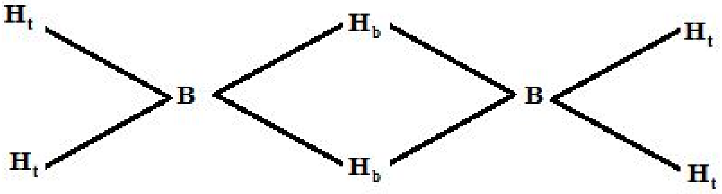
OSF_2 has pyramida/ shape.

138. The bonding in diborane $\left(B_2H_6\right)$ can be described by

1) 4 two centre - two electron bonds & 2 three - centre - two electron bonds
2) 3 two centre - two electron bonds & 3 three centre - two electron bonds
3) 2 two centre - two electron bonds and 4 three centre - two electron bonds
4) 4 two centre - two electron bonds and 4 two centre - two electron bonds

Key: 1

Sol: Diborane molecular formula is B_2H_6



Diborane has 4 two centered two e^- bonds and two three centered two electron bonds.

139. The monomers of Buna - S rubber are

1) Isoprene and butadiene	2) Butadiene and phenol
3) Styrene and butadiene	4) Vinyl chloride and sulphur

Key: 3

Sol: The monomers present in buna- S rubber are

(1) 1, 3- Butadiene

(2) styrene

140. Heating a mixture of Cu_2O and Cu_2S will give

1) $CuO + CuS$	2) $Cu + SO_3$	3) $Cu + SO_2$	4) $Cu\left(OH\right)_2 + CuSO_4$

Key: 3

Sol: Heating a mixture of $Cu_2O \& Cu_2S$ will give $Cu \& SO_2$.

141. Which of the following corresponds to the energy of the possible excited state of hydrogen?

1) - 13.6 eV	2) 13.6 eV	3) - 3.4 eV	4) 3.4 eV

Key: 3

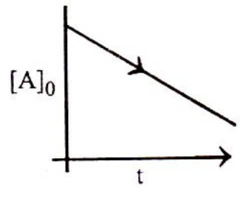
Sol: $En = -\frac{13.6}{n^2} eV$

$= -\frac{13.6}{2^2} eV$

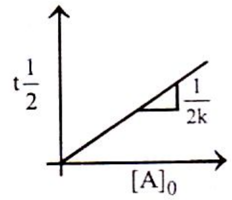
$= -\frac{13.6}{4} eV$

$= -3.4 eV \cdot$

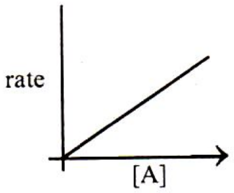
142. Which of the following are the correct representations of a zero order reaction, where A represents the reactant?



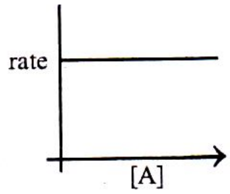
(a)



(b)



(c)



(d)

1) a, b, c 2) a, b, d 3) b, c, d 4) a, c, d

Key: 2

Sol: In zero order R'n

$t\frac{1}{2} \propto \frac{[A]_0}{2K}$

143. The set representing the right order of ionic radius is

1) $Li^+ > Na^+ > Mg^{2+} > Be^{2+}$ 2) $Mg^{2+} > Be^{2+} > Li^+ > Na^+$

3) $Na^+ > Mg^{2+} > Li^+ > Be^{2+}$ 4) $Na^+ > Li^+ > Mg^{2+} > Be^{2+}$

Key: 4

Sol: The correct order of Ionic radius is

$Na^+ > Li^+ > Mg^{+2} > Be^{+2}$

144. Which one of the following statement is correct for d^4 ions [P=pairing energy]

1) When $\Delta_0 >$ P, low-spin complex form

2) When $\Delta_0 <$ P, low-spin complex form

3) When $\Delta_0 >$ P, high-spin complex form

4) When $\Delta_0 <$ P, both high-spin and low-complexes form

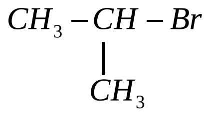
Key: 1

Sol: When $\Delta_0 >$ pairing energy pairing of electrons takes place

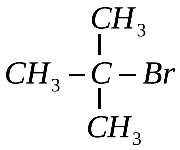
$\therefore$ Low- spin complexes are formed

145. The reactivity of alkyl bromides.

1) CH_3CH_2Br

2)


$CH_3 - \underset{\displaystyle CH_3}{\underset{|}{CH}} - Br$

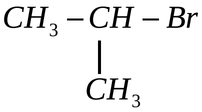
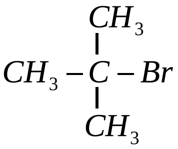
3)


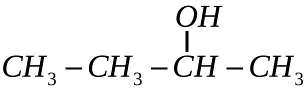
$CH_3 - \overset{\displaystyle CH_3}{\overset{|}{\underset{\displaystyle CH_3}{\underset{|}{C}}}} - Br$

4) CH_3Br

Key: 1

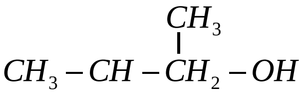
Sol: In dry acetone reaction follows SN^2 mechanism

Hence $CH_3Br > CH_3CH_2Br >$  $\underset{CH_3}{\underset{|}{CH_3-CH-Br}}$ $>$  $CH_3-\overset{CH_3}{\underset{CH_3}{C}}-Br$

146. Optically active  $CH_3-CH_3-\overset{OH}{\overset{|}{C}}H-CH_3$ **was found to have lost its optical activity after standing in water containing a few drops of acid, mainly due to the formation of**

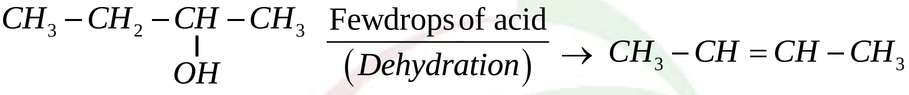
1) $CH_3-CH_2-CH=CH_2$

2) $CH_3-CH=CH-CH_3$

3)  $CH_3-CH-\overset{CH_3}{\overset{|}{C}}H_2-OH$

4) $CH_3-CH_2-CH_2-OH$

Key: 2

Sol: 

$$\underset{OH}{\underset{|}{CH_3-CH_2-CH-CH_3}}\ \frac{\text{Fewdrops of acid}}{(Dehydration)}\rightarrow CH_3-CH=CH-CH_3$$

147. Commercially available H_2SO_4 is 98 gms by weight of H_2SO_4 and 2 gms by weight of water. It's density is 1.83 g cm^{-3}. Calculate the molality (m) of H_2SO_4 (molar mass of H_2SO_4 is 98 g mol^{-1})

1) 500 m
2) 20 molal
3) 50 m
4) 200 m

Key: 1

Sol: $m=\frac{w_{solute}}{GMW_{solute}}\times\frac{1000}{W_{solvent}}=\frac{98}{98}\times\frac{1000}{2}=500m$

148. Cylohexylamine and aniline can be distinguished by

1) Hinsberg test
2) Carbylamine test
3) Lassaigne test
4) Azo dye test

Key: 4

Sol: Cyclohexylamine is not the aromatic amine so do not give azodye test

149. ________ is a potent vasodilator.

1) Histamine
2) Serotonin
3) Codeine
4) Cimetidine

Key: 1

Sol: Histamine is a potent vasodilator.

150. Standard Enthalpy (Heat) of formation of liquid water at 25^0C is around

$$H_{2(g)}+\frac{1}{2}O_{2(g)}\rightarrow H_2O_{(l)}$$

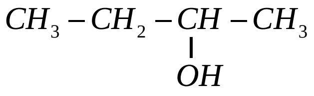
1) -237 KJ/mol
2) 237 kJ/mol
3) -286 kJ/mol
4) 286 kJ/mol

Key: 3

Sol: $\Delta H^0{}_{f[H_2o(l)]}=-286\,kJ/mol$

151. The alcohol that reacts faster with Lucas reagent is

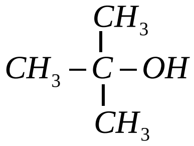
1) $CH_3-CH_2-CH_2-CH_2-OH$

2) 

$CH_3-CH_2-\underset{OH}{\underset{|}{CH}}-CH_3$

3) 

$CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-OH$

4) 

$CH_3-\overset{CH_3}{\overset{|}{\underset{CH_3}{\underset{|}{C}}}}-OH$

Key: 4

Sol: Tertiary alcohols are more reactie towards lucas reagent

152. Balance the following equation by choosing the correct option

$\text{xKnO}_3 + yC_{12}H_{22}O_{11} \rightarrow pN_2 + qCO_2 + rH_2O + sK_2CO_3$

	x	y	p	q	r	s
1)	36	55	24	24	5	48
2)	48	5	24	36	55	24
3)	24	24	36	55	48	5
4)	24	48	36	24	5	55

Key: 2

Sol: Verify from options

153. Which of the following elements is purified by vapour phase refining?

1) Fe
2) Zr
3) Cu
4) Au

Key: 2

Sol: Zr is easily decomposed in vapour phase,

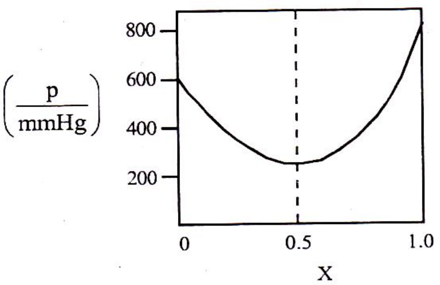
154. When helium gas is allowed to expand in to vaccum, heating effect is observed. The reason for this is (Assume He as a non ideal gas)

1) He is an inert gas
2) The inversion temperature of Helium is very high
3) The inversion temperature of Helium is very low
4) He has the lowest boiling point

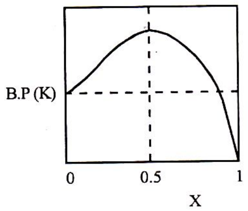
Key: 3

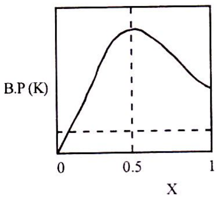
Sol: Gases with low inversion temperature gets heated when allowed to expand in vaccum

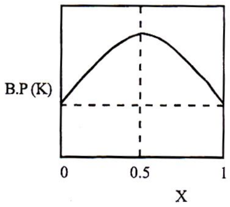
155. The vapour pressure of a non- ideal two component solution is given below

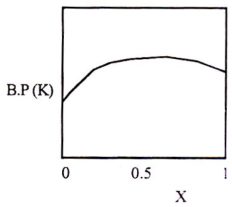


Identify the correct T-X curve for the same mixture.

1) 

2) 

3) 

4) 

Key: 1

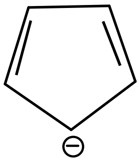
Sol: Vapour pressure is inversely proportional to boiling point.

156. Cyclopentadienyl anion is

1) benzenoid and aromatic
2) non-benzenoid and aromatic
3) non-benzenoid and non-aromatic
4) non-benzenoid and anti-aromatic

Key: 2

Sol:



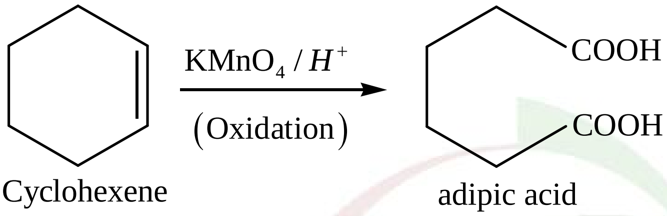
Cyclopentadienyl anion It is cyclic, planar with $\left(4n+2\right)\pi e^{-}$ There fore aromatic but structure is not similar to benzene

157. Oxidation of cyclohexene in presence of acidic potassioum permanganate leads to

1) glutaric acid
2) adipic acid
3) pimelic acid
4) succinic acid

Key: 2

Sol:



158. How many emission spectral lines aer possible when hydrogen atom is excited to nth energy level?

1) $\frac{n\left(n+1\right)}{2}$
2) $\frac{\left(n+1\right)}{2}$
3) $\frac{\left(n-1\right)n}{2}$
4) $\frac{n^2}{4}$

Key: 3

Sol: Conceptual

159. The bond length (pm) of F_2, H_2, Cl_2 *and* I_2, **respectively is**

1) 144, 74, 199, 267
2) 74,144, 199, 267
3) 74, 267, 199, 144
4) 144.74.267.199

Key: 1

Sol: Order of bond length is $\text{H}_2 < F_2 < Cl_2 < I_2$

160. The number of tetrahedral and octahedral voids in CCP unit cell are respectively

1) 4, 8
2) 8,4
3) 12, 6
4) 6, 12

Key: 2

Sol: If there are n atoms in a CCP, the number of tetrahedral voids are 2n and octahedral voids are n.

* * *