# SRIGAYATRI EDUCATIONAL INSTITUTIONS

**INDIA**

**SR MPC** **29-03-2020**

## JEE MAINS MODEL

## SECTION – I
## (SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.
**Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.**

## MATHS

**SYLLABUS: Matrices (inverse and linear equations), Differentiation**

**1.** **If** $A+B+C=\pi,$ **then the value of the determinant** $D=\begin{vmatrix}\sin^2 A & \cot A & 1\\ \sin^2 B & \cot B & 1\\ \sin^2 C & \cot C & 1\end{vmatrix}$ **is:**

A) 1 B) -1 C) 0 D) None of these

**2.** **A divisor of the determinant** $D=\begin{vmatrix}a^2+1 & ab & ac\\ ba & b^2+1 & bc\\ ca & cb & c^2+1\end{vmatrix}$ **is:**

A) $1+a^2+b^2+c^2$ B) $a^2+b^2+c^2$ C) $(a+b+c)^2$ D) $ab+bc+ca$

**3.** **If** $\alpha, \beta$ **and** $\gamma$ **are the roots of the equation** $x^3+px+q=0,$ **then the value of the determinant** $\begin{vmatrix}\alpha & \beta & \gamma\\ \beta & \gamma & \alpha\\ \gamma & \alpha & \beta\end{vmatrix}=$

A) $p$ B) $q$ C) $p^2-2q$ D) None of these

**4.** **The determinant** $\begin{vmatrix}\cos(\theta-\phi) & -\sin(\theta+\phi) & \cos 2\phi\\ \sin\theta & \cos\theta & \sin\phi\\ -\cos\theta & \sin\theta & \cos\phi\end{vmatrix}$ **is:**

A) 0 B) Independent of $\theta$
C) Independent of $\phi$ D) Independent of $\theta$ and $\phi$ both

**5.** **The determinant** $\begin{vmatrix}\cos(x-y) & \cos(y-z) & \cos(z-x)\\ \cos(x+y) & \cos(y+z) & \cos(z+x)\\ \sin(x+y) & \sin(y+z) & \sin(z+x)\end{vmatrix}$ **is:**

A) $2\sin(x-y)\sin(y-z)\sin(z-x)$ B) $-2\sin(x-y)\sin(y-z)\sin(z-x)$
C) $2\cos(x-y)\cos(y-z)\cos(z-x)$ D) $-2\cos(x-y)\cos(y-z)\cos(z-x)$

**6.** **The determinant** $\begin{vmatrix}b+c & a & a\\ b & c+a & b\\ c & c & a+b\end{vmatrix}$ **is:**

A) $2abc$ B) $3abc$ C) $4abc$ D) None of these

**7.** **The set of homogeneous equations** $tx+(t+1)y+(t-1)z=0$, $(t+1)x+ty+(t+2)z=0$, $(t-1)x+(t+2)y+tz=0$ **has non-trivial solution for:**

A) Three values of $t$ B) Two values of $t$ C) One value of $t$ D) No value of $t$

**8.** **If** $a, b, c$ **are real then the value of determinant** $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1$ **if:**

A) $a+b+c=0$ B) $a+b+c=1$ C) $a+b+c=-1$ D) $a=b=c=0$

**9.** **The equation** $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

A) Has no real solution
B) Has 4 real solutions
C) Has two real and two non-real solutions
D) Has infinite number of solutions, real or non-real

**10.** **The solution of the following equations:**
$x-(4k-3)y+2z=0,\ kx-(2k-1)y+(k+1)z=0,\ (2k+2)x+3ky+(k+2)z=0$ **For** $k=1$, $(x,y,z)$ **is given by:**

A) $(t,3t,t)$ B) $(-2t,-10t,-4t)$ C) $(9t,-5t,-7t)$ D) $(4t,6t,t)$

**11.** **If** $(\sin x)(\cos y)=\frac{1}{2}$ **then** $\frac{d^2y}{dx^2}$ **at** $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ **is**

A) – 4 B) – 2 C) – 6 D) 0

**12.** **If** $t(1+x^2)=x$ **and** $x^2+t^2=y$ **then at** $x=2, \frac{dy}{dx}=$

A) $\frac{24}{5}$ B) $\frac{101}{125}$ C) $\frac{488}{125}$ D) $\frac{358}{125}$

**13.** **If** $f(x)=|x-2|$ **and** $g(x)=f[f(x)]$ **then** $\sum_{r=0}^{3} g'(2r-1)$ **is equal to**

A) 0 B) 1 C) – 1 D) 2

**14.** **If** $y=\frac{1}{x}$ **then** $\frac{dy}{\sqrt{1+y^4}}+\frac{dx}{\sqrt{1+x^4}}=$

A) 0 B) 1 C) $x/y$ D) $y/x$

**15.** **If** $x=\frac{1+t}{t^3}$ **and** $y=\frac{3+4t}{2t^2}$ **then** $x(y')^3=$

A) $1-y'$ B) $1+y$ C) $y'-1$ D) $y'$

**16.** **The value of** $y''(1)$**, if** $x^3-2x^2y^2+5x+y-5=0$ **when** $y(1)=1$ **is equal to**

A) $\frac{22}{7}$ B) $-7\frac{21}{28}$ C) 8 D) $-8\frac{22}{27}$

**17.** **If** $f'(3)=2$ **then** $\lim_{x\to\infty}\frac{f(3+h^2)-f(3-h^2)}{2h^2}$ **is**

A) 1 B) 2 C) 3 D) 1/2

**18.** **If** $f(x)=\frac{\tan x+\sec x-1}{\tan x-\sec x+1}$ **then** $f'(x)$ **is equal to**

A) $\sec x(\tan x-\sec x)$ B) $\sec x(\sec x-\tan x)$ C) $\sec x(\sec x+\tan x)$ D) None of these

**19.** **If** $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ **, then** $\frac{dy}{dx}$ **is equal to**

A) $2\sec^2(x/2)$ B) $(1/2)\sec^2(x/2)$ C) $(1/2)$ D) $-(1/2)\sec^2(x/2)$

**20.** $f(x) = \begin{cases} |x-4| & for\ x \geq 1 \\ \frac{x^3}{2} - x^2 + 3x + 1/2 & for\ x < 1 \end{cases}$ **then**

A) $f(x)$ is continuous at $x = 1$ and $x = 4$ B) $f(x)$ is differentiable at $x = 4$

C) $f(x)$ is continuous and differentiable at $x = 1$ D) $f(x)$ is only continuous at $x = 1$

## SECTION-II
## (Numerical Value Answer Type)

**21.** **Let** $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$ **and suppose that** $\det(A) = 2$ **then the** $\det(B)$ **equals, where**

$B = \begin{bmatrix} 4x & 2a & -p \\ 4y & 2b & -q \\ 4z & 2c & -r \end{bmatrix}$

**22.** **Let** $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$, **then the maximum value of** $f(x)$ **is:**

**23.** **Let** $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$ **then** $f\left(\frac{\pi}{6}\right)$ **is:**

**24.** **If** $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right), -\pi < x < \pi$ **then** $\frac{dy}{dx} =$

**25.** **Let** $y = x^3 - 8x + 7$ **and** $x = f(t)$. **If** $\frac{dy}{dt} = 2$ **and** $x = 3\ at\ t = 0$ **then the value of** $\frac{dx}{dt}$ **at t = 0 _____**

## SECTION – I
## (SINGLE CORRECT ANSWER TYPE)

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## PHYSICS

## SYLLABUS: Rotational Motion, Gravitation

**26.** **Moment of inertia of solid sphere about its diameter is I. If that sphere is recast into 8 identical small spheres, then the moment of inertia of small sphere about its diameter is:**

A) $\frac{I}{8}$ B) $\frac{I}{16}$ C) $\frac{I}{24}$ D) $\frac{I}{32}$

**27.** **Three particles each of mass ‘m’ are arranged at the corner of an equilateral triangle of side ‘L’. If one of the masses is doubled, the shift in the centre of mass of the system**

A) $\frac{L}{\sqrt{3}}$ B) $\frac{L}{4\sqrt{3}}$ C) $\frac{\sqrt{3}L}{4}$ D) $\frac{L}{2\sqrt{3}}$

**28. A uniform thin bar of mass 6m and length 12L is bent to make a regular hexagon. Its moment of inertia about an axis passing through centre of mass and perpendicular to the plane of the hexagon is:**

A) $20\,mL^2$ B) $6\,mL^2$ C) $\frac{12}{5}mL^2$ D) $30\,mL^2$

**29. A thin rod of length L and mass M is held vertically with one end on the floor and is allowed to fall. Find the velocity of the other end when it hits the floor, assuming that the end on the floor does not slip**

A) $\sqrt{\frac{3g}{L}}$ B) $\sqrt{3gL}$ C) $\sqrt{\frac{L}{3g}}$ D) $\sqrt{\frac{g}{3L}}$

**30. A cylinder of mass M, radius R is resting on a horizontal platform (which is parallel to XY plane) with its axis fixed along the Y axis and free to rotate about its axis. The platform is given a motion in X - direction given by $x = A\cos\omega t$. There is no slipping between cylinder and platform. The maximum torque acting on the cylinder during its motion is:**

A) $\frac{1}{2}MRA\omega^2$ B) $MRA\omega^2$ C) $2MRA\omega^2$ D) $2MRA\omega^2\cos wt$

**31. Find the gravitational attraction on mass m due to the quarter ring of same mass m and Radius 'R' as shown in figure.**


A) $\frac{2\sqrt{2}Gm^2}{\pi R^2}$ B) $\frac{2Gm^2}{\pi R^2}$ C) $\frac{Gm^2}{\pi R^2}$ D) $\frac{\sqrt{2}Gm^2}{\pi R^2}$

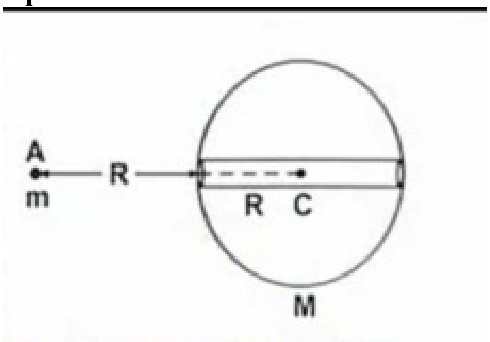
**32. If diameter of a planet is four times that of earth. What is the time period of a pendulum on that planet, if it is a second pendulum on earth. Take mean density of the planet equal to that of earth.**

A) 4 sec B) 1 sec C) 2 sec D) 3 sec

**33. Two particles of masses $m_1$ and $m_2$ are initially at rest at infinite separation. When released, what is their relative velocity of approach when they are at a separation d.**

A) $\sqrt{\frac{G(m_1+m_2)}{2d}}$ B) $\sqrt{\frac{G(m_1+m_2)}{d}}$ C) $\sqrt{\frac{2G(m_1+m_2)}{d}}$ D) $\sqrt[2]{\frac{2G(m_1+m_2)}{d}}$

**34. Figure shows a fixed solid sphere of mass M and radius R with a narrow smooth tunnel along its diameter. Another particle of mass m is placed at point A as shown. If the particle is released from rest, then its speed when it reaches the center of sphere is**




A) $\sqrt{\frac{GM}{R}}$ B) $\sqrt{\frac{2GM}{R}}$ C) $\sqrt{\frac{GM}{2R}}$ D) $2\sqrt{\frac{GM}{R}}$

**35. Two satellites A and B of same mass are orbiting earth at altitudes R and 3R respectively, where R is Radius. The earth taking their orbits to be circular, find the ratio of their kinetic energies.**

A) 1 : 3 B) 1 : 1 C) 2 : 1 D) 3 : 1

**36. A Satellite of mass $2\times10^3$ kg is to be shifted from on orbit of radius 2R to another orbit of radius 3R. Calculate the minimum energy required for this. [R is the Radius of earth]** $\left[g=10ms^{-2}\right]$

A) $1.066\times10^{10}J$ B) $1.066\times10^{9}J$ C) $2.132\times10^{10}J$ D) $2.132\times10^{9}J$

**37. An artificial satellite is moving in a circular orbit around earth with a speed equal to half the escape Velocity from earth surface. Find the height of satellite above the earth surface.**

A) 3200 Km B) 6400 Km C) 10800 Km D) 19600 Km

**38. The radius of a planet is $R_1$ and a satellite revolves around it in a circular orbit of radius $R_2$. The period of revolution is T. What is the acceleration due to gravity on the surface of the planet?**

A) $\frac{4\pi^2R_1^3}{R_2^2T^2}$ B) $\frac{\pi^2R_1^3}{R_1T}$ C) $\frac{4\pi^2R_2^3}{R_1^2T^2}$ D) $\frac{\pi^2R_1^3}{R_2^2T^2}$

**39. A satellite revolves around a planet in an elliptical orbit. Its perigee and apogee are $0.5\times10^7\ m$ and $1.5\times10^7\ m$ respectively. If the minimum speed of satellite in orbit is $5\times10^3\ m/s$, then its maximum speed in orbit is**

A) $15\times10^4\ m/s$ B) $\frac{5}{3}\times10^3\ m/s$ C) $1.5\times10^4\ m/s$ D) $\frac{5}{3}\times10^4\ m/s$

**40. The change in the value of 'g' at a height 'h' above the surface of earth is same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct?**

A) $d=\frac{h}{2}$ B) $d=\frac{3h}{2}$ C) $d=2h$ D) $d=h$

**41. The distance of the centres of moon and earth is d. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitional field will be zero.**

A) $\frac{d}{2}$ B) $\frac{2d}{3}$ C) $\frac{4d}{3}$ D) $\frac{9d}{10}$

**42. A point P lies on the axis of a ring of mass M and radius R at a distance R from its centre. A small particle starts from p and reaches the centre under gravitational attraction only. It's speed at center will be**

A) $\sqrt{\frac{2GM}{R}}$ B) $\sqrt{\frac{2GM}{R}\left[1-\frac{1}{\sqrt{2}}\right]}$ C) Zero D) $\sqrt{\frac{GM}{R}}$

**43. The gravitational field in a region is given by $\overline{E}=(4i+j)\,N/Kg$. Work done by this field is Zero when a particle is moved along the line is**

A) $y+4x=2$ B) $4y+x=6$ C) $x+y=5$ D) All of these

**44. Two masses 90 Kg and 160 Kg are at a distance 5 m apart. Find the magnitude of intensity of the gravitational field at a point which is at a distance 3 m from 90 Kg and 4 m from 160 Kg mass.**

A) $10\sqrt{2}\ G\ N\ Kg^{-1}$ B) $10\ G\ N\ Kg^{-1}$ C) $7\ G\ N\ Kg^{-1}$ D) $2\ G\ N\ Kg^{-1}$

**45. Find the height from the earth's surface where acceleration due to gravity is 25% of it's value on the surface of earth. [R=6400Km]**

A) 3200 Km B) 6400 Km C) 10800 Km D) 19600 Km

## SECTION- II
## (Numerical Value Answer Type)

**46. Two kids weighing 10 kg and 15 kg respectively, are trying to balance a see saw of total length 5.00 m, with it's fulcrum at the centre. If one of the kids is sitting at an end, then what is the distance in meter from the fulcrum to the other kid _____**

**47. A small hole is made in a disc of mass M and radius R at a distance $\frac{R}{4}$ from center. The is supported on supported on a horizontal peg through this hole. The moment of inertia of the disc about the peg is $\frac{9}{a}$.The value of a is _____**

**48. A force of 490 N acts tangentially at the highest point of a sphere of mass 5 kg kept on a horizontal plane. If the sphere rolls without slipping, then the acceleration of center of sphere in $m/s^2$ is _____**

**49. A solid sphere is projected up along an inclined plane of inclination $\theta = 30^0$ with a speed $V = 2ms^{-1}$.If it rolls without slipping, then the maximum distance traversed by it in m is $(g = 10ms^{-2})$**

**50. A child is standing with folded hands at the center of a platform rotating about is central axis. The K.E of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The K.E of the system is now BK. The value of B is _____**

## SECTION – I
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## CHEMISTRY

**SYLLABUS: I-A, II-A, III-A &IV-A Groups, Junior organic Chemistry (GOC) and Alkanes**

**51. Find out 'Z' in the following sequence of reaction** $Na \xrightarrow{Air} x \xrightarrow{Maisture\ in\ air} y \xrightarrow{Cu} z$

1) $Na_2CO_3$ 2) $NaOH$ 3) $Na_2O_2$ 4) $Na_2O$

**52. Aluminum reacts with $NaOH$ and forms compound 'X' If the co-ordination number of Al in 'X' is 6,the correct formula, of X is**

1) $\left[Al(H_2O)_4(OH)_2\right]^+$ 2) $\left[Al(H_2O)_3(OH)_3\right]$ 3) $\left[Al(H_2O)_2(OH)_4\right]^-$ 4) $\left[Al(H_2O)_6\right](OH)_3$

**53. Amongst $LiCl, RbCl, BeCl_2$ and $MgCl_2$ the compounds with the greatest and the least ionic character, respectively are :**

1) $LiCl$ and $RbCl$ 2) $RbCl$ and $BeCl_2$
3) $MgCl_2$ and $BeCl_2$ 4) $RbCl$ and $MgCl_2$

**54. Magnesium burns in air giving two products. Their forumulae are**

1) $MgO, MgN$ 2) $MgO_2, Mg_3N_2$ 3) $MgO, Mg_3N_2$ 4) $Mg_2O, Mg_3N_2$

**55. A piece of magnesium is burnt in air to give ash. Then ash is dissolved in water to liberated a colorless gas, which turns red litmus to blue.The colorless gas is**

1) $NH_3$ 2) $NO_2$ 3) $N_2$ 4) $NO$

**56. Sedimentary rocks laid down under water mainly contain**

1) $CaO$ 2) $Ca(OH)_2$ 3) $CaCO_3$ 4) $CaSO_4$

**57. The property not associated with refractory nature of Mg is**

1) it has high m.p
2) it is a good conductor of heat
3) it is chemically inert as well as electrical insulator
4) forms Grignard's reagent

**58.** $BCl_3 + LiAIH_4 \rightarrow A + LiCI + AICI_3$

$A + H_2O \rightarrow B + H_2,\ B \xrightarrow{Red\ heat} C$. **In this raction sequence A,B and C compounds respectively are**

1) $B_2H_6, B_2O_2, B$ 2) $B_2H_6, H_3BO_3, B_2O_3$ 3) $B_2H_6, H_3BO_3, B$ 4) $HBF_{4,}H_3BO_3, B_2O_3$

**59. Which amongst the following is also called as a sesqui oxide**

1) $B_2O_3$ 2) $Al_2O_3$ 3) $Tl_2O_3$ 4) all

**60. Which of the following is a lewis acid**

1) $AlCl_3$ 2) $MgCl_2$ 3) $CaCl_2$ 4) $BaCl_2$

**61. Silica reacts with hydride of super halogen to form.'X' on hydrolysis of X another compound Y on heating to form Z.The 'Z' can also be prepared in the following reaction.**

1) $SiO_2 + 4HF \rightarrow SiF_4 + 2H_2O$ 2) $Si + O_2 \rightarrow SiO_2$
3) $K_2SiF_6 + 4K \rightarrow 6KF + Si$ 4) $Si + 2NaOH + H_2O \rightarrow Na_2SiO_3 + 2H_2O$

**62. Which glass has the highest percentage of lead?**

1) Soda glass 2) Flint glass 3) Jena glass 4) Payrex glass

**63. Addition of sodium hydroxide solution to a weak acid (HA) results in a buffer of $P^H$ is 6. If ionisation constant of HA is $10^{-5}$,the ratio of salt to acid concentration in the buffer solution will be**

1) 5:4 2) 1:10 3) 4:5 4) 10:1

**64. Decreasing order of 'P' Character in the following**

**A :** $SiO_2; B: CO_2; C:$Graphite

1) $A > B > C$ 2) $B > A > C$ 3) $B > C > A$ 4) $A > C > B$

**65. The difference in properties of $CH_4$ and $SiH_4$ is due to**

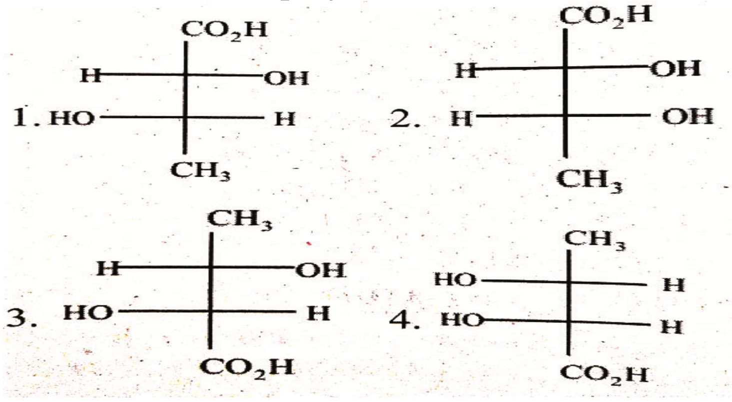
1) Large difference in the electronegativity of carbon and silicon
2) Less difference in the size of carbon and Silicon atoms
3) The inability of carbon to expand its octet
4) The inability of silicon to form double bonds

**66. The IUPAC name of the alkane is**

**is**

1) 2- Isopropyl-2, 6, 6-trimethylheptane
2) 5 tert-butyl-2isopropyl-2-methypentane
3) 2, 3, 3, 7, 7-Pentamethyloctane
4) 2, 2, 6, 6, 7-Pentamethyloctane

**67. The correct Fischer projection formula of (2R,3R)-2,3-dihydroxy butanoic acid is**

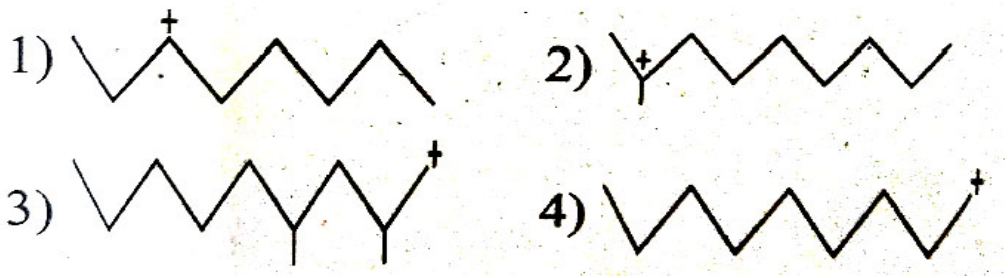


**68. Among the following, the true property**

$$\underset{CH_3}{\overset{CH_3}{>}}\overset{+}{C}-CH_3$$ **is about**

1) non-planar
2) $C^+$ is sp2-hybridized
3) electrophile can attack $C^+$
4) does not undergo hydrolysis

**69. Select the most stable carbocation from amongst the following**



**70. Polarization of electrons in acrolein may be written as**

1) $\overset{\delta-}{C}H_2 = CH - \overset{\delta+}{C}H = O$
2) $\overset{\delta-}{C}H_2 = CH - CH = \overset{\delta+}{O}$
3) $\overset{\delta-}{C}H_2 = \overset{\delta-}{C}H - CH = O$
4) $\overset{\delta-}{C}H_2 = CH - CH = \overset{\delta+}{O}$

## NUMERICAL VALUE QUESTIONS

**71. How many of the following elements of I A group as lighter than water (i) Cs (ii) Na (iii) K (iv) Li**

**72. How many of the following will not give any colour in flame (i) Be (ii) Mg (iii) Na (iv) Li**

**73. Magnesium oxide when mixed with a saturated solution of magnisium chloride sets to a hard mass like cement known as "sorel cement". The composition of Sorel cement is . What is the value of n___**

**74. On heating calcium ammoniate, ammonia and hydrogen are evolved. How many moles of ammonia are evolved when 1.5 moles of calcium ammoniate are heated___**

**75. The ratio of copper and tin in bell metal alloy is ______.**

# SRIGAYATRI EDUCATIONAL INSTITUTIONS

INDIA

SR MPC

JEE MAINS MODEL

29-03-2020

## KEY SHEET

### MATHS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1) **3** | 2) **1** | 3) **4** | 4) **2** | 5) **2** | 6) **3** | 7) **3** | 8) **4** | 9) **4** | 10) **3** |
| 11) **1** | 12) **3** | 13) **1** | 14) **1** | 15) **2** | 16) **4** | 17) **1** | 18) **3** | 19) **2** | 20) **1** |
| 21) **-16** | 22) **6** | 23) **1** | 24) **0.5** | 25) **0.11** | | | | | |

### PHYSICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 26) **4** | 27) **2** | 28) **1** | 29) **2** | 30) **1** | 31) **1** | 32) **2** | 33) **3** | 34) **2** | 35) **3** |
| 36) **1** | 37) **2** | 38) **3** | 39) **3** | 40) **3** | 41) **4** | 42) **2** | 43) **1** | 44) **1** | 45) **2** |
| 46) **1.67** | 47) **16** | 48) **140** | 49) **0.56** | 50) **0.5** | | | | | |

### CHEMISTRY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 51) **1** | 52) **3** | 53) **2** | 54) **3** | 55) **1** | 56) **3** | 57) **4** | 58) **2** | 59) **4** | 60) **1** |
| 61) **2** | 62) **2** | 63) **4** | 64) **4** | 65) **1** | 66) **4** | 67) **2** | 68) **2** | 69) **2** | 70) **4** |
| 71) **3** | 72) **2** | 73) **5** | 74) **8** | 75) **4** | | | | | |

# SRIGAYATRI EDUCATIONAL INSTITUTIONS

INDIA

SR MPC

**JEE MAINS MODEL**

29-03-2020

## HINTS & SOLUTIONS

## MATHS

1. Use: $R_3 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$
2. Multiply $R_1$ 2y $a$, $R_2$ 2y $b$ and $R_3$ 2y $c$ and divide the determinant 2y $abc$. Now take $a, b$ and $c$ common from $c_1, c_2$ and $c_3$. Now use $C_1 \to C_1 + C_2 + C_3$ to get $\left(a^2+b^2+c^2+1\right)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

   Now use: $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$ to get the value as 1.
3. $D = \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$

   Suppose given $\alpha + \beta + \gamma = 0$

   As coefficient of $x^2 = 0$

   Now, $\det\left(\alpha+\beta+\gamma\right)\begin{vmatrix} 1 & 1 & 1 \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$
4. Directly open 2y $R_1$ to get

   $\cos^2\left(\theta+\phi\right) + \sin^2\left(\theta+\phi\right) + \cos 2\phi = 1 + \cos 2\phi$. Which is independent of $\theta$
5. Expand the determinant using first row and use $x - y = A,\ y - z = B$ and $z - x = C$

   $\Rightarrow A + B + C = 0$
6. Put $a = b = c = 1$.
7. D simplifies to $-4\left(2t+1\right) = 0$
8. Multiply $R_1$ 2y a, $R_2$ 2y 2 and $R_3$ 2y c and divide the determinant 2y $abc$. Now take $a, b$ and $c$ common from $c_1, c_2$ and $c_3$. Now use $C_1 \to C_1 + C_2 + C_3$ to get:

   $$\left(a^2+b^2+c^2+1\right)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = 1$$

   Now use $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

   We get $1 + a^2 + b^2 + c^2 = 1$

   $\Rightarrow a = b = c = 0$
9. 1st two columns of 1st determinant are same as 1st two rows of 2nd. Hence transpose the 2nd. Add the two determinants and use $C_1 \to C_1 + C_3 \Rightarrow D = 0$
10. Put $k = 1$ and solve
11. $\sin x.\cos y = \dfrac{1}{2}$

    $\Rightarrow \cos x.\cos y - \sin y.\sin x \dfrac{dy}{dx} = 0$

    $\Rightarrow \dfrac{dy}{dx} = \cot x.\cot y$

$\Rightarrow \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x.\cot y - \operatorname{cosec}^2 y.\cot x.\frac{dy}{dx}$

Now $\left(\frac{dy}{dx}\right)_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)} = 1$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)} = -2(1) - 2(1)(1) = -4$

12. Given $t = \frac{x}{1+x^2} \Rightarrow \frac{dt}{dx} = \frac{1-x^2}{\left(1+x^2\right)^2}$

$x^2 + t^2 = y \Rightarrow \frac{dy}{dx} = 2x\left[\frac{\left(1+x^2\right)^3 + 1 - x^2}{1-x^4}\right]$

$\therefore \frac{dy}{dx} = \frac{2x\left[\left(1+x^2\right)^3 + 1 - x^2\right]}{\left(1+x^2\right)^3}$

13. $g(x) = \left||x-2|-2\right| = \begin{cases} |x|, & x \le 2 \\ 4-x, & 2 < x \le 4 \\ x-4, & 4 < x \end{cases}$

$g'(-1) + g'(1) + g'(3) + g'(5) = -1 + 1 - 1 + 1 = 0$

14. $\frac{dy}{dx} = -1/x^2$

$\frac{dy}{dx} + \frac{\sqrt{1+y^4}}{\sqrt{1+x^4}} = \frac{dy}{dx} + \frac{1}{x^2} = -\frac{1}{x^2} + \frac{1}{x^2} = 0$

15. $\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$

16. Differentiating the given expression, we get $3x^2 - 4xy^2 - 4x^2y.y' + 5 + y' = 0$

at $x = 1$, we have $3 - 4.1.1 - 4.1.1.y' + 5 + y'(1) = 0$

$4 - 3y'(1) = 0 \Rightarrow y'(1) = 4/3$

Differentiating again, we have $6x - 4y^2 - 8xy.y' - 8xy.y' - 4x^2(y')^2 - 4x^2y.y'' + y'' = 0$

Putting $x = 1,\ y = 1$ *and* $y'(1) = 4/3$, we get $6 - 4 - 8\left(\frac{4}{3}\right) - 8\left(\frac{4}{3}\right) - 4\left(\frac{16}{9}\right) - 3y''(1) = 0$

$y''(1) = -8.\frac{22}{7}$

17. $\lim_{y\to 0} \frac{f\left(3+h^2\right) - f\left(3-h^2\right)}{2h^2}$

$= \frac{1}{2}\lim_{h\to 0} \frac{f\left(3+h^2\right) - f(3)}{h^2}$

$= \frac{1}{2} f'(3) = 2/2 = 1$

18. $f(x) = \frac{\tan^2 x + \sec^2 x + 2\tan\sec x - 1}{(1+\tan x)^2 - \sec^2 x}$

$= \frac{\tan^2 x + \sec^2 x + 2\tan x \sec - 1}{2\tan x} = \tan x + \sec x$

Thus $f'(x) = \sec^2 x + \sec x \tan x = \sec x(\sec x + \tan x)$

19. $y = \tan^{-1}\sqrt{\frac{2\sin^2 x/2}{2\cos^2 x/2}} = \tan^{-1}\tan(x/2) = x/2$

Hence $\frac{dy}{dx} = \frac{1}{2}$

20. Since $g(x) = |x|$ is continuous function and $\lim_{h\to 1+} f(x) = 3\lim_{h\to 1-} f(x)$ so far f is a continuous function. In a particular f is continuous at $x = 1$ and $x = 4$. $f$ is clearly not differentia2le at $x = 4$. Since $g(x) = |x|$ is not differentia2le at $x = 0$. Now

$f'(1+) = \lim_{h\to 0+} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h\to 0+} \frac{|-3+h| - 3}{h} = -1$

$f'(1-) = \lim_{h\to 0+} \frac{(1/2)(1+h)^3 - (1+h)^2 + 3(1+h) + 1/2 - 3}{h}$

$f'(1-) = \lim_{h\to 0-} \frac{(1/2)(1+h)^3 - (1+h)^2 - (h^2+2h) + 3h}{h} = 5/2$

Hence f is not differentia2le at $x = 1$

21. $\det(B) = \begin{vmatrix} 4x & 2a & -p \\ 4y & 2b & -q \\ 4z & 2c & -r \end{vmatrix} = (4)(2)(-1)\begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$

$= -8\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = -8\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

$= -8 \times 2 = -16$

22. Use: $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$ and expand to get $f(x) = 2 + 4\sin 2x$.

23. Put $\theta = \frac{\pi}{6}$ and find determinant.

24. $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

$y = \tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

$y = \tan^{-1}\left(\tan\frac{x}{2}\right)$

$y = \frac{x}{2} \qquad \left(\therefore -\pi < x < \pi \; or \; \frac{-\pi}{2} < \frac{x}{2} < \frac{\pi}{2}\right)$

25. $y = x^3 - 8x + 7$

$\frac{dy}{dx} = 3x^2 - 8$

$t = 0 \Rightarrow x = 3$

$$\frac{dy}{dx}=3(3)^2-8=19$$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2}{19}=0.1053$$

## PHYSICS

26. $I=MR^2$

$M\ \alpha\ R^3$

$R\ \alpha\ M^{1/3}$

$$\frac{R_2}{R_1}=\left(\frac{M_2}{M_1}\right)^{1/3}$$

$$\frac{I_2}{I_1}=\frac{M_2}{M_1}\left(\frac{R_2}{R_1}\right)^2$$

$$\frac{I_2}{I_1}=\frac{M_2}{M_1}\left(\frac{M_2}{M_1}\right)^{2/3}$$

$$I_2=\frac{I_1}{32}$$

27.

$$x_{cm}=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$$

And also $y_{cm}$

If one of masses dou2led find $x'_{cm}, y'_{cm}$

$$r=\sqrt{\left(x_{cm}-x'_{cm}\right)^2+\left(y_{cm}-y'_{cm}\right)^2}$$

28. $I=6\,I_{oneside}$

$$=6\left[\frac{m(2L)^2}{12}+mr^2\right]$$

Where $r=\sqrt{3}l$

$I=20mL^2$

29. $Mg\frac{L}{2}=\frac{1}{2}I\omega^2$

$$Mg\frac{L}{2}=\frac{1}{2}\frac{ML^2}{3}\omega^2$$

$$\omega=\sqrt{\frac{3g}{L}}$$

$V=r\omega$

$V=\sqrt{3gL}$

30. $x=A\cos\omega t$

$$V=\frac{dx}{dt}=-A\omega\sin\omega t$$

$a=-A\omega^2\cos\omega t$

$a_{max}=A\omega^2$

$T_{max}=I\,\alpha_{max}$

$$= I\frac{a_{\max}}{R}$$

$$= \frac{1}{2}MR^2 \times \frac{A\omega^2}{R}$$

$$T_{\max} = \frac{1}{2}MAR\omega^2$$

31. Mass of element of ring

$$dm = \frac{2m}{\pi}d\theta$$

Force on 'm' due to dm

$$dF = \frac{Gmdm}{R^2}$$

$$dF = \frac{2Gm^2 d\theta}{\pi R^2}$$

Net force on m will 2e

$$F = \int_{\frac{-\pi}{4}}^{\frac{+\pi}{4}} dF\cos\theta$$

$$F = \frac{2\sqrt{2}Gm^2}{\pi R^2}$$

32. $g = \frac{GM}{R^2} = \frac{G\rho\frac{4}{3}\pi R^3}{R^2}$

$$g = G\rho\frac{4}{3}\pi R$$

$$T\alpha\frac{1}{\sqrt{g}}$$

$g\alpha R$ $\quad \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{R_2}{R_1}}$

$$\frac{2}{T_2} = \sqrt{\frac{4R_1}{R_1}}$$

$$T_2 = 1\sec ond$$

33. At separation 'd'

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_1^2 = \frac{Gm_1m_2}{d}$$

From law of conservation of linear momentum

$$m_1v_1 = m_2v_2$$

From (1) and (2)

$V_1 = \sqrt{\frac{2GM_2^2}{d(m_1+m_2)}}$ and

$$V_2 = \sqrt{\frac{2GM_1^2}{d(m_1+m_2)}}$$

Relative velocity

$$V_{rel} = V_1 + V_2$$

34. potential due to sphere at point $A, V_A = \frac{-GM}{2R}$

Potential due to sphere at point $C, V_c = \frac{-3}{2}\frac{GM}{R}$

Work done $W = +\left[V_A - V_C\right] \to (1)$

$W = \frac{1}{2}mv^2 \to (2)$

From (1) and (2) $V = \sqrt{\frac{2GM}{R}}$

35. Or2ital speed, $V = \sqrt{\frac{GM}{R+h}}$

Kinetic energy = $\frac{1}{2}mv^2$

$$\Rightarrow \frac{KE_1}{KE_1} = \left[\frac{V_1}{V_2}\right]^2 = \frac{R+h_2}{R+h_1} = \frac{R+3R}{R+R} = \frac{4R}{2R} = \frac{2}{1}$$

36. Total energy of a satellite, $E = \frac{-GMm}{2r}$

Work done in shifting, $W = E_f - E_i$

$$W = +\frac{GMm}{2R}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$W = +\frac{gRm}{12}$$

$$W = \frac{10 \times 6400 \times 10^3 \times 2 \times 10^3}{12}$$

$$W = 1.066 \times 10^{10} J$$

37. Or2it speed $V_0 = \sqrt{\frac{GM}{R+h}}$

Given $V_0 = \frac{1}{2}V_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$

$$\Rightarrow \sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \frac{1}{R+h} = \frac{1}{2R}$$

$$\Rightarrow h = r = 6400 Km$$

38. Time period, $T^2 = \frac{4\pi^2 R_2^3}{GM}$

Acceleration due to gravity

$$g = \frac{GM}{R_1^2}$$

2ut

$$GM = \frac{4\pi^2 R_2^3}{T^2}$$

$$\Rightarrow g = \frac{4\pi^2 R_2^3}{R_1^2 T^2}$$

39. From conservation of angular momentum

$L_a = L_p$

$mr_1v_1 = mr_2v_2$

$V_1 = \frac{r_2v_2}{r_1} = \frac{1.5\times10^7\times5\times10^3}{0.5\times10^7}$

$1.5\times10^4 m/s$

40. For smaller heights,

$g_n = g\left[1-\frac{2h}{R}\right]$

Change in g, $\Delta g_1 = g - g_n$

$=\frac{2h}{R}$

At depth d,

$g_d = g\left[1-\frac{d}{R}\right]$

Change in g, $\Delta g_2 = g - gd$

$=\frac{d}{R}$

$\Delta g_1 = \Delta g_2$

Given $\frac{2h}{R} = \frac{d}{R}$

$\Rightarrow d = 2h$

41.

$\frac{GM_e}{x^2} = \frac{GM_m}{(d-x)^2}$

$\frac{81M_m}{x^2} = \frac{M_m}{(d-x)^2}$

$\frac{9}{x} = \frac{1}{d-x}$

$x = \frac{9d}{10}$

42. Potential at a distance R

$V_p = -\frac{GM}{\sqrt{R^2+R^2}} = -\frac{GM}{\sqrt{2R}}$

Potential at the centre

$V_p = -\frac{GM}{R}$

Work done $W = -\left[V_p - V_C\right]m \to (1)$

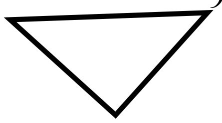
And $W = \frac{1}{2}mv^2 \to (2)$

From (1) and (2), $V = \sqrt{\frac{2GM}{R}\left[1-\frac{1}{\sqrt{2}}\right]}$

43. Work done on a particle is zero, if force and displacement are perpendicular.

44. Due to 90 Kg, at'p'

$E_1 = \frac{G(90)}{3^2} = 10G$




Due to 160 Kg,

$E_2 = \frac{G(160)}{4^2} = 10G$

Resultant field,

$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2} \cos 90^0$

$E = 10\sqrt{2} GNKg^{-1}$

45. At a height 'h'

$g_n = \frac{g}{\left[1+\frac{h}{R}\right]^2}$ ; $g_h = \frac{g}{4}$

$\Rightarrow 4 = \left[1+\frac{h}{R}\right]^2$

$\Rightarrow 2 = 1+\frac{h}{R}$

$\Rightarrow h = R = 6400Km$

46. In rotational equili2rium

Torque due to

One kid a2out fulcrum = Torque due to other kid a2out fulcrum

$m_1 g r_1 = m_2 g r_2$

$10 \times 2.5 = 15 \times x$

$x = \frac{2.5 \times 2}{3} = \frac{5}{3} = 1.67m$

47. From parallel axes theorem

$I = I_0 + Mr^2 ; r = \frac{R}{4}$

$I = \frac{MR^2}{2} + \frac{MR^2}{16}$

$I = \frac{9MR^2}{16} = \frac{9MR^2}{a}$

48. acceleration, $a = \frac{10F}{7F}$

$= \frac{10 \times 490}{7 \times 5}$

$= 140ms^{-2}$

49. From conservation of energy

$$\frac{1}{2}mv^2\left[1+\frac{K^2}{R^2}\right]=mgh$$

$$h=\frac{V^2\left[1+\frac{K^2}{R^2}\right]}{2g}=\frac{4\left[1+\frac{2}{5}\right]}{20}=\frac{7}{5}$$

$$\sin\theta=\frac{h}{1}$$

Distance,

$$l=\frac{h}{\sin\theta}$$

$$=\frac{h}{\sin 30^0}$$

$$=\frac{7}{25}\times\frac{2}{1}$$

$$=\frac{14}{25}$$

$$=0.56m$$

50. Angular momentum is constantss

$$K.E=\frac{L^2}{2I}$$

$$K.E\alpha\frac{1}{I}$$

$$\frac{KE_1}{KE_1}=\frac{I_2}{I_1}=\frac{2I_1}{I_1}$$

$$KE_2=\frac{KE_1}{2}$$

$$=0.5KE_1$$