SRIGAYATRI EDUCATIONAL INSTITUTIONS

INDIA

SR MPC | **JEE MAINS MODEL (UT-2)** | **Time: 3:00 Hrs.**

Date: 24-04-2020 | **Max. Marks: 300 M**

SECTION-I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking Scheme: +4 for correct answer, 0 if not attempted and -1 if not correct

MATHEMATICS

SYLLABUS: TRIGONOMETRY, COMPLEX NUMBERS, DE MOIVERSES THEOREM

1. **In** ΔABC, **if** $\cot A\cot B\cot C = k$, **then**_____

1) $k \le \frac{1}{3\sqrt{3}}$ 2) $k \ge \frac{1}{3\sqrt{3}}$ 3) $k < \frac{1}{9}$ 4) $k > \frac{1}{3}$

2. **If** $\theta = 3\alpha$ **and** $\sin\theta = \frac{3}{5}$. **Then the value of** $3\text{cosec}\alpha - 4\sec\alpha$ **is**_____

1) 0 2) $\frac{9}{2}$ 3) 10 4) 24

3. **If** $\cos^2 A + \cos^2 B + \cos^2 C = 1$, **then** ΔABC **is** _____

1) Equilateral 2) isosceles 3) right angled 4) right angled isosceles

4. **The ratio** $\frac{\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} - \sin\frac{6\pi}{7}}{\sin\frac{\pi}{7}\sin\frac{3\pi}{7}\sin\frac{5\pi}{7}} =$ _____

1) $\frac{1}{4}$ 2) 1 3) 2 4) 4

5. **If the equation** $x^2 + 4x\sin\theta + \tan\theta = 0$ **(where** $0 < \theta < \frac{\pi}{2}$**) has repeated roots. Then** $\theta =$ _____

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{12}$ *or* $\frac{5\pi}{12}$ 4) $\frac{\pi}{6}$ *or* $\frac{\pi}{12}$

6. $\sin^4\theta - (k+2)\sin^2\theta - (k+3) = 0$ **admits a solution if** $k \in$ __

1) $[0,\infty]$ 2) $[-3,-2]$ 3) $[-\infty,0]$ 4) $\left[-\sqrt{3},\sqrt{3}\right]$

7. **If** $[\sin\theta + 1] + [\cos\theta + 1] = 0$. **Then quadrant in which** θ **lies is** _____
(where [ ] is greatest integer function).

1) 1 2) 2 3) 3 4) 4

8. **The sum of all solutions of the equation** $\cot\theta = \sin 2\theta\left(\theta \in [0,\pi]\right)$ **is** _____

1) $\frac{3\pi}{2}$ 2) π 3) $\frac{3\pi}{4}$ 4) 2π

9. $\sin^{-1}(\sin 3) + \sin^{-1}(\sin 4) + \sin^{-1}(\sin 5) =$ ___

1) -1 2) -2 3) 12 4) $2-\pi$

10. $\sin^{-1}\left(\cos\sin^{-1}x\right) + \cos^{-1}\left(\sin x\cos^{-1}x\right) =$ _____

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{3\pi}{4}$

11. **If** $\tan^{-1}\left(4^{\cos^2\theta}+1\right)+\cot^{-1}\left(6+2\cos\theta+\cos^2\theta\right)=\frac{\pi}{2}$. **Then** _____

1) $\cos\theta=1$ 2) $\cos\theta=-1$ 3) $\cos\theta=\frac{-1}{2}$ 4) all the above

12. **In** ΔABC, **with usual notation, if** $2\left(2ab+ac-a^2\right)=4b^2+c^2$, **Then** $\sin B=$ ___

1) $\frac{\sqrt{15}}{8}$ 2) $\frac{\sqrt{3}}{5}$ 3) $\frac{3}{8}$ 4) $\frac{7}{11}$

13. **In** ΔABC, **with usual notation, if** $\cos A+\sin A=\frac{2}{\cos B+\sin B}$, **then** $\left(\frac{a+b}{c}\right)^2=?$

1) 2 2) 1 3) 3 4) 12

14. **Two vertical poles 20 m and 60 m stands a part on a horizontal plane. The height of the point of intersection of the lines joining the top of each pole to the foot of the other is_____**

1) 15 m 2) 16 m 3) 18 m 4) 50 m

15. **If** $|z-1|=1$, **Then** $Arg\ z=$ _____

1) $\frac{\pi}{4}$ 2) $\frac{1}{2}Arg\left(z+1\right)$ 3) $\frac{1}{2}Arg\left(z-1\right)$ 4) $\frac{\pi}{2}$

16. **If** $x=i\left(i+\sqrt{2}\right)$, **Then** $x^4+4x^3+6x^2+4x+3=$ ____

1) 6 2) -6 3) 0 4) 9

17. **If** z_1,z_2,z_3 **are 3 complex numbers in A.P; then they lie on _____**

1) a circle 2) a straight line 3) a parabola 4) an ellipse

18. **Let** $1,\alpha_1,\alpha_2,...\alpha_{n-1}$ **are the nth roots of unity, then** $\left(2-\alpha_1\right)\left(2-\alpha_2\right)....\left(2-\alpha_{n-1}\right)=?$

1) 2^n-1 2) 2^n+1 3) 2^n 4) n

19. **If** $\left|z^2-3\right|=3|z|$, **then maximum value of** $|z|$ **is** _____

1) 1 2) $\frac{3+\sqrt{21}}{2}$ 3) $\frac{-3+\sqrt{21}}{2}$ 4) $\frac{21-\sqrt{3}}{2}$

20. **Least positive argument of the 4th root of the complex number** $2-i\sqrt{12}$ **is** _____

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{12}$ 3) $\frac{5\pi}{12}$ 4) $\frac{7\pi}{12}$

SECTION-II
(Numerical Value Answer Type)

This section contains 5 questions. The answer to each question is a Numerical values comprising of positive or negative decimal numbers.

Marking Scheme: +4 for correct answer, 0 in all other cases.

21. **Sum of squares of all trigonometrical ratios has minimum value as ____**

22. **If roots of** $x^3-66x^2+165x-1=0$ **be sides of** ΔABC, **Then** $\sum\left(\frac{\cos A}{a}\right)=$ _____

23. **The value of** $\cos\left(\tan^{-1}1+\tan^{-1}2+\tan^{-1}3\right)$ **is ____**

24. **If** $\left(5x^3+Mx+N\right)$ **is perfectly divisible by** $\left(x^2+x+1\right)$, **Then M + N = _____**

25. **If z satisfies** $\bar{z}+z=0$ **and** $|z|^2-4zi=z^2$, **then** $|z|=$ ____

SECTION-I
(SINGLE CORRECT ANSWER TYPE)

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PHYSICS

SYLLABUS: GRAVITATION, OSCILLATIONS AND WAVES

26. The time period of mass loaded spring is 'T'. If 20% of its turns are cut and removed, the same mass is attached to the remaining spring would oscillates with a time period of

1) $\frac{T}{\sqrt{5}}$ 2) $\frac{2T}{\sqrt{5}}$ 3) $\frac{T}{5\sqrt{2}}$ 4) $\sqrt{2}\,T$

27. A pendulum is taken 1km inside the earth from sea level. Then the gain or loss of time period of the pendulum in second is

1) 2 2) 4 3) 7 4) 9

28. A simple pendulum has a time period T_1, when on the earth's surface T_2 when it is taken a height 'R' above the earths surface, where R is the radius of the earth. The value of $\frac{T_2}{T_1}$ is

1) 2 2) 4 3) 6 4) 8

29. Two pendulums of length 16L and L are in phase at the mean position at a certain time. If T is the time period of shorter pendulum, the minimum time after which they will be again in phase is

1) $\frac{2}{3}T$ 2) $\frac{2}{5}T$ 3) $\frac{4}{3}T$ 4) $\frac{4}{5}T$

30. When a body of mass 1kg is suspended from a certain light spring hanging vertically. Its length increases by 5cm. By suspending 2kg block to the spring and if the block is pulled through 10cm and released the maximum velocity in it in m/sec is $\left(g = 10 m/\sec^2\right)$

1) 1 2) 2 3) 3 4) 4

31. If the radius of the earth is 6400km, the height above the surface of the earth, where the acceleration due to gravity value will be 1% of its value from the surface of the earth is

1) 3R 2) 5R 3) 7R 4) 9R

32. If g is the acceleration due to gravity on the earth and R is the radius of the earth and angular speed of rotation of the earth about its axis is made to be $\sqrt{\frac{2g}{5R}}$. Then weight of a body at equator decreases by

1) 10% 2) 40% 3) 70% 4) 100%

33. The energy required to shift a satellite from orbital radius r to radius 2r is E. What energy will be required to shift the satellite from orbital radius 2r to 3r

1) $\frac{E}{2}$ 2) $\frac{E}{3}$ 3) $\frac{E}{4}$ 4) E

34. Ratio of the radius of the planet A to that of planet B is 20. The ratio of acceleration due to gravity on the surface of the two planets is 5. The ratio of the escape velocity of the two planets is

1) 5 2) 10 3) 15 4) 20

35. The altitude at which the weight of a body is 36% less than its weight on the surface of the earth value is

1) 6400km 2) 3200km 3) 1600km 4) 400km

36. The wave velocity of a progressive wave is 480 m/s and the phase difference between the two particles separated by distance of 12m is 1080^0. The number of waves passing across a point in 1sec is

1) 120 2) 240 3) 160 4) 360

37. When the temperature of an ideal gas is increased by 600k, the velocity of sound in the gas becomes $\sqrt{3}$ times the initial velocity in it. The initial temperature of the gas is

1) -73^0C 2) 27^0C 3) 127^0C 4) 327^0C

38. A wave of frequency 200Hz is sent along a string towards a fixed point (end). When this wave travels back then after reflection, a node is formed at a minimum distance of 5cm from the fixed end of the string. The speed of the incident wave is

1) 5 m/s 2) 10 m/s 3) 20 m/s 4) 40 m/s

39. A source of sound producing wavelength 50 cm is moving away from a stationary observer with 1/5th. Speed of sound. Then what is the wavelength of sound heard by the observer

1) 60 cm 2) 70 cm 3) 40 cm 4) 55 cm

40. The successive resonance frequencies in an open organ pipe are 1944 Hz and 2600 Hz. The length of the pipe of the speed of sound in air is 328 m/s

1) 0.04 m 2) 0.25 m 3) 0.4 m 4) 0.5 m

41. The maximum particle velocity is 3 times the wave velocity of a progressive wave. If the amplitude of the particle is 'a'. The phase difference between the two particles separated by a distance *'x'* is

1) x/a 2) $3x/a$ 3) $3a/x$ 4) $3\pi x/a$

42. A metallic wire with tension T and temperature 30^{0}C vibrates with it fundamental frequency of 1K.Hz . The same wire with the same tension but at 10^{0}C temperature vibrates with a fundamental frequency of 1.001 KHz. The coefficient of linear expansion of the wire is

1) $1\times10^{-4}C^{-1}$ 2) $0.5\times10^{-4}C^{-1}$ 3) $1.5\times10^{-4}C^{-1}$ 4) $2\times10^{-4}C^{-1}$

43. Two stationary sources A and B produces sounds of same frequency. A person running from A to B hears 6 beats/sec. If the frequency of each source increases by 100Hz then 8 beats/sec are heard. Then the original frequency of each source is

1) 100 Hz 2) 150 Hz 3) 200 Hz 4) 300 Hz

44. Two different sound sources S_1 and S_2 have frequencies ratio 1:2, source S_1 is approaching towards an observer and S_2 is receding from the same observer. Speeds of both S_1 and S_2 are the same and equal to V. Speed of sound in air is 300 m/s. If no beats are heard by the observer the value of V is

1) 50 m/s 2) 75 m/s 3) 100 m/s 4) 125 m/s

45. A whistle of frequency 540 Hz rotates in a horizontal circle of radius 2m at an angular speed of 15 rad/s. The highest frequency heard by a listener at rest with respect to the centre of circle (velocity of sound in air is 330 m/s)

1) 509 Hz 2) 594 Hz 3) 598 Hz 4) 602 Hz

SECTION-II
(Numerical Value Answer Type)

46. A body executing S.H.M has a maximum acceleration equal to 48 m/sec^2 and a maximum velocity equal to 12 m/sec. The amplitude of S.H.M in metre is_____

47. A geostationary satellite orbits around the earth in a circular orbit which is at height of 36000 km from the surface of the earth. Then the period of spy satellite orbiting a few hundred km above the earth surface in hour is____ $(R_{earth} = 6400\,km)$

48. Two satellite of masses 100 kg and 200 kg are revolving the earth altitudes 6400 km and 44800km. The ratio of orbital velocity of satellite is

49. The frequency of a stretched uniform wire under tension is in resonance with the fundamental frequency of a closed tube. If the tension in the wire is increased by 8N. It is in resonance with the first over tone of the closed tube . The initial tension in wire_____N.

50. Two identical flutes produces fundamental nodes of frequency 300 Hz at 27^{0}C. If the temperature of air in one flute increases to 31^{0}C the number of beats per second will be _____

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(SINGLE CORRECT ANSWER TYPE)

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CHEMISTRY

SYLLABUS: SECOND YEAR INORGANIC CHEMISTRY

51. A metal 'x' on heating in nitrogen gas gives Y. Y on treatment with H_2O gives a colourless gas which when passed through $CuSO_4$ solution gives a blue colour 'Y' is

1) $Mg(NO_3)_2$ 2) Mg_3N_2 3) NH_3 4) MgO

52. Extra pure Nitrogen can be obtained by heating

1) $NH_3 with\ CuO$ 2) NH_3NO_3 3) $(NH_4)_2Cr_2O_7$ 4) $Ba(N_3)_2$

53. Among the following oxyacid acid of phosphorous which contains p-p bond is

1) $H_4P_2O_7$ 2) $H_4P_2O_5$ 3) $H_3P_3O_9$ 4) $H_4P_2O_6$

54. Which of the following are incorrect statements

1) solid PCl_5 exists as tetrahedral $(PCl_4)^+$ and octahedral $(PCl_6)^-$ ions

2) solid PBr_5 exists as $(PBr_4)^+ Br^-$

3) solid N_2O_5 exists as $NO_2^+NO_3^-$

4) PCl_6^- is trigonal bipyramidal in shape

55. Which of the following statement is true

1) both rhombic and monoclinic sulphur are soluble in water

2) both rhombic and monoclinic sulphur are soluble in CS_2

3) both rhombic and monoclinic sulphur are insoluble in CS_2

4) Rhombic sulphur can be converted in to monoclinic sulphur but the reverse is not possible

56. Which of the following statement is wrong

1) H_2S is a dibasic acid 2) H_2S acts only as a reducing agent

3) the bond angle is H_2S is 109^028^1 4) H_2S has rotten egg smell

57. Which of the following statements regarding sulphur is incorrect ?

1) S_2 is para magnetic

2) the vapour at 200^0C consists mostly S_8 rings

3) At 600^0C, the gas mainly consists S_2 molecules

4) the oxidation state of sulphur is never less than +4 in its compounds

58. Which of the following acids contain peroxy linkage

I) oleum **II) marshall's acid** **III) caro's acid** **IV) sulphuric acid**

1) I & II only 2) II & III only 3) I, II & IV only 4) II, III and IV only

59.

$$\underset{(X_1)}{\text{Orange solid}} \xrightarrow{\Delta} \underset{(gas)}{\text{diamagnetic}} + \underset{(residue)}{\text{green}}$$

$$\underset{solid(X_2)}{\text{Orange}} \xrightarrow{\Delta} \underset{(\text{gas})}{\text{paramagnetic}} + \underset{(\text{residue})}{\text{green}} + \underset{(solid)}{\text{yellow}}$$

X_1 and X_2 are respectively:

1) $Na_2Cr_2O_7,\ K_2Cr_2O_7$ 2) $(NH_4)_2Cr_2O_7,\ Na_2Cr_2O_7$

3) $K_2Cr_2O_7,\ (NH_4)_2Cr_2O_7$ 4) $(NH_4)_2Cr_2O_7,\ KNO_3$

60. In the following reaction sequence in aqueous solution, the species X,Y and Z respectively are

$S_2O_3^{2-} \xrightarrow{Ag^+} \underset{(Clear\ sol.)}{X} \xrightarrow{Ag^+} \underset{(White)}{Y\downarrow} \xrightarrow{with\ time} \underset{(Black)}{Z\downarrow}$

1) $\left[Ag(S_2O_3)_2\right]^{3-}, Ag_2S_2O_3, Ag_2S$ 2) $\left[Ag(S_2O_3)_3\right]^{5-}, Ag_2SO_3, Ag_2S$

3) $\left[Ag(SO_3)_2\right]^{3-}, Ag_2S_2O_3, Ag$ 4) $\left[Ag(SO_3)_3\right]^{3-}, Ag_2SO_4, Ag$

61. Which of the following is correctly arranged in order of increasing hydration energy ?

1) $ClO_4^- < ClO_3^- < ClO^- < ClO_2^-$ 2) $ClO_4^- < ClO^- < ClO_2^- < ClO_2^-$

3) $ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$ 4) $ClO^- < ClO_2^- < ClO_3^- < ClO_4^-$

62. Which of the following statements are correct ?

1) the repulsion between the two lone pairs of electrons of the oxygen atom of F_2O reduces the F-O-F bond angle from 109^0 to 105^0

2) In both Cl_2O and Br_2O molecules, the bond angles are greater than that of the OF_2 molecule because of steric crowding of the larger halogen atoms (Cl and Br)

3) Cl_2O, Br_2O and OF_2 all are bent molecules

4) the O-F bond length in OF_2 is longer than in O_2F_2

63. Which pair gives Cl_2 at room temperature ?

1) Conc HCl+$KMnO_4$ 2) NaCl+Conc H_2SO_4 3) NaCl+MnO_2 4) NaCl+Conc HNO_3

64. Match the items of Columns I and II and mark the correct option.

Column – I	Column – I
A. Its partial hydrolysis does not change Oxidation state of central atom	1. He
B. It is used in modern diving apparatus	2. XeF_6
C. It is used to provide inert atmosphere for ? lling electrical bulbs	3. XeF_4
D. Its central atom is in sp3d3 hybridisation.	4. Ar

	A	B	C	D		A	B	C	D
1)	1	4	2	3	2)	1	2	3	4
3)	2	1	4	3	4)	1	3	2	4

65. Which of the following diagrams is correct related to Xe ?

1) $\boxed{Xe(g)} \xrightarrow[(1:20\ ratio)]{F_2} \boxed{XeF_6} \xrightarrow{Hydrolysis} \boxed{Xe}$

2) $\boxed{\begin{matrix}Xe(g)\\ Excess\end{matrix}} \xrightarrow[(2:1\ ratio)]{F_2} \boxed{XeF_2} \xrightarrow{Hydrolysis} \boxed{Xe}$

3) $\boxed{Xe(g)} \xrightarrow[(1:20\ ratio)]{F_2} \boxed{XeF_6} \xrightarrow{SiO_2} \boxed{Xe}$

4) all are correct

66. Many transition metals form interstitial compounds. The characteristics of these interstitial compounds are

I) They have low melting points **II) They are very hard**

III) They retain metallic conductivity **IV) They are chemically more reactive than the pure metals.**

1) II, III only correct 2) I, III only correct 3) II, IV only correct 4) IV only correct

67. When copper ore is mixed with silica, in a reverberatory furnace copper matte is produced. The copper matte contains:

1) sulphides of copper (II) and iron (II) 2) sulphides of copper (II) and iron (III)

3) sulphides of copper (I) and iron (II) 4) sulphides of copper (I) and iron (III)

68. Actinoids exhibit more number of oxidation states in general than lanthanoids. This is because

1) the 5f orbitals are more buried than the 4f orbitals
2) there is a similarity between 4f and 5f orbitals in their angular part of the wave function
3) actinoids are more reactive than lanthanoids
4) the 5f orbitals extend farther from the nucleus than the 4f orbitals

69. One mole of the complex compound $Co(NH_3)_5Cl_3$ gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of AgCl (s). The structure of complex is

1) $\left[Co\left(NH_3\right)_4 Cl\right]Cl_2, NH_3$
2) $\left[Co\left(NH_3\right)_5 Cl\right]Cl_2$
3) $\left[Co\left(NH_3\right)_3 Cl_3\right]2.NH_3$
4) $\left[Co\left(NH_3\right)_4 Cl_2\right]Cl.NH_3$

70. $\left[Ni\left(CN\right)_4\right]^{2-}, \left[MnBr_4\right]^{2-}$ and $\left[FeF_6\right]^{3-}$. Geometry, hybridization and magnetic moment of the ions respectively are

1) Tetrahedral, square planar, octahedral: $sp^3, dsp^2, sp^3d^2 : 5.9, 0, 5.9$
2) Tetrahedral, square planar, octahedral: $dsp^2, sp^3, sp^3d^2 : 0, 5.9, 5.9$
3) Square planar, tetrahedral, octahedral: $dsp^2, sp^3, d^2sp^3 : 5.9, 5.9, 0$
4) Square planar, tetrahedral, octahedral: $dsp^2, sp^3, sp^3d^2 : 0, 5.9, 5.9$

SECTION-II
(Numerical Value Answer Type)

71. How many of the following are diamagnetic ?

$\left[Ag\left(NH_3\right)_2\right]^{+1}, \left[Cd\left(NH_3\right)_4\right]^{2+}, Cr\left(CO\right)_6, \left[Ni\left(NH_3\right)_6\right]^{2+}, \left[Ni\left(CN\right)_4\right]^{2-}$

72. How many of the following have d^2sp^3 hybridization ?

$\left[Ni\left(NH_3\right)_6\right]^{2+}, \left[Co\left(NH_3\right)_6\right]^{3+}, \left[Fe\left(CN\right)_6\right]^{4-}, \left[Cr\left(NH_3\right)_6\right]^{3+} \left[Co\left(en\right)_3\right]^{3+}, \left[Co\left(EDTA\right)\right]^{2-},$

$\left[CoF_6\right]^{3-}, \left[Co\left(H_2O\right)_6\right]^{2+}$

73. The number of stereoisomers possible in $Ma_2b_2c_2$ are

74. A molten mixture contain following substance, Cu, FeO, PbO, CaO, MgO, Ag
Find the total no.of substances which are removed when molten mixture is treated with SiO_2.

75. The number of isomers possible for $\left[Ir\left(Cl\right)\left(Br\right)\left(NH_3\right)\left(py\right)\right]$ is

KEY SHEET

MATHS

1) **1**	2) **3**	3) **3**	4) **4**	5) **3**	6) **2**	7) **3**	8) **1**	9) **2**	10) **3**
11) **2**	12) **1**	13) **1**	14) **2**	15) **3**	16) **1**	17) **2**	18) **1**	19) **2**	20) **3**
21) **7**	22) **2013**	23) **-1**	24) **-5**	25) **2**					

PHYSICS

1) **2**	2) **3**	3) **1**	4) **3**	5) **1**	6) **4**	7) **2**	8) **3**	9) **2**	10) **3**
11) **1**	12) **2**	13) **3**	14) **1**	15) **2**	16) **2**	17) **1**	18) **4**	19) **3**	20) **2**
21) **3**	22) **1**	23) **2**	24) **1**	25) **2**					

CHEMISTRY

1) **2**	2) **4**	3) **4**	4) **4**	5) **3**	6) **3**	7) **4**	8) **2**	9) **2**	10) **1**
11) **3**	12) **4**	13) **1**	14) **3**	15) **2**	16) **1**	17) **3**	18) **4**	19) **2**	20) **4**
21) **4**	22) **5**	23) **6**	24) **4**	25) **3**					

HINTS & SOLUTIONS

MATHS

1. In ΔABC, we have $\sum \tan A = \pi \tan A$

Now $AM \geq GM \Rightarrow \dfrac{\tan A + \tan B + \tan C}{3} \geq \left(\tan A \tan B \tan C\right)^{\frac{1}{3}}$

$\Rightarrow \tan A \tan B \tan C \geq 3\left(\tan A \tan B \tan C\right)^{\frac{1}{3}}$

$\Rightarrow \tan A \tan B \tan C \geq 3^{\frac{3}{2}}$

$\Rightarrow \dfrac{1}{k} \geq 3\sqrt{3} \Rightarrow k \leq \dfrac{1}{3\sqrt{3}}$

2. $\text{Data} \Rightarrow \sin 3\alpha = \dfrac{3}{5} \Rightarrow \cos 3\alpha = \dfrac{4}{5}$

$\text{Now } 3\operatorname{cosec}\alpha - 4\sec\alpha = \dfrac{3}{\sin\alpha} - \dfrac{4}{\cos\alpha}$

$= \dfrac{3\cos\alpha - 4\sin\alpha}{\sin\alpha\cos\alpha}$

$= \dfrac{5}{\sin\alpha\cos\alpha}\left(\dfrac{3}{5}\cos\alpha - \dfrac{4}{5}\sin\alpha\right)$

$= \dfrac{5}{\sin\alpha\cos\alpha}\left(\sin 3\alpha\cos\alpha - \cos 3\alpha\sin\alpha\right)$

$= \dfrac{10}{2\sin\alpha\cos\alpha}\left(\sin\left(3\alpha - \alpha\right)\right)$

$= \dfrac{10}{\sin 2\alpha}\left(\sin 2\alpha\right) = 10$

3. $\text{Data} \Rightarrow \cos^2 A + \cos^2 B - \left(1 - \cos^2 C\right) = 0$

$\Rightarrow \cos^2 A + \cos^2 B - \sin^2 C = 0$

$\Rightarrow \cos^2 A + \cos\left(B + C\right)\cos\left(B - C\right) = 0$

$\Rightarrow 2\cos A\cos B\cos C = 0$

$\Rightarrow \text{either } A = 90^0 \text{ or } B = 90^0 \text{ or } C = 90^0$

$\therefore \Delta ABC$ is right angled triangle

4. Let N,D denotes numerator, denominator. Thus

$N = \sin 2\theta + \sin 4\theta - \sin 6\theta$

$D = \sin 2\theta\ \sin 3\theta\ \sin 5\theta$

$\text{Now } N = \left(\sin 2\theta + \sin 4\theta\right) - \sin 6\theta$

$= 2\sin 3\theta\cos\theta - 2\sin 3\theta\cos 3\theta$

$= 2\sin 3\theta\left(\cos\theta - \cos 3\theta\right)$

$= 4\sin\theta\ \sin 2\theta\ \sin 3\theta$

$\text{But } \sin 2\theta = \sin\dfrac{2\pi}{7} = \sin\left(\pi - \dfrac{5\pi}{7}\right) = \sin\dfrac{5\pi}{7} = \sin 5\theta$

Thus $N = 4\sin\theta\sin 3\theta\sin 5\theta.$

$\therefore \frac{N}{D} = \frac{4\sin\theta\sin 3\theta\sin 5\theta}{\sin\theta\sin 3\theta\sin 5\theta} = 4.$

5. By data, Quadratic has equal roots

$\Rightarrow \Delta = 0 \Rightarrow 16\sin^2\theta - 4\tan\theta = 0$

$\Rightarrow 4\sin^2\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \sin 2\theta = \frac{1}{2}$

$\Rightarrow 2\theta = \frac{\pi}{6}\ (or)\ \frac{5\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{12}\ (or)\ \frac{5\pi}{12}$

6. $\sin^2\theta = \frac{(k+2) \pm \sqrt{(k+2)^2 + 4(k+3)}}{2}$

$= \frac{(k+2) \pm (k+4)}{2} = -1, (k+3)$

Now $\sin^2\theta = (k+3)^2$ *as* $\sin^2\theta = -1$

not possible

But $0 \le \sin^2\theta \le 1 \Rightarrow 0 \le k+3 \le 1$

$\Rightarrow -3 \le k \le -2$

7. Data $\Rightarrow [\sin\theta] + [\cos\theta] = -2$

Above relation is true in 3rd quadrant because sin, cos are –ve in 3rd quadrant.

8. Data $\Rightarrow \frac{\cos\theta}{\sin\theta} = 2\sin\theta\cos\theta$

$\Rightarrow \cos\theta(1 - 2\sin^2\theta) = 0$

$\Rightarrow \cos\theta = 0$ *or* $\sin\theta = \pm\frac{1}{\sqrt{2}}$

$(where\ 0 < \theta < \pi)$

$\Rightarrow \left(\theta = \frac{\pi}{2}\right), \left(\theta = \frac{\pi}{4}, \frac{3\pi}{4}\right)$

sum of solutions $= \frac{3\pi}{2}.$

9. We get $\sin^{-1}(\sin 3) = \pi - 3$

$\sin^{-1}(\sin 4) = \pi - 4$

$\sin^{-1}(\sin 5) = 5 - 2\pi$

Adding, we get sum = -2

10. We have $\cos(\sin^{-1}x) = \sqrt{1-x^2}$

$\sin(\cos^{-1}x) = \sqrt{1-x^2}$

Now given expression $= \sin^{-1}\sqrt{1-x^2} + \cos^{-1}\sqrt{1-x^2} = \frac{\pi}{2}$

11. By substitution, observe that $\cos\theta = -1$ satisfies given equation

12. On simplifying, $(a-c)^2 + (a-2b)^2 = 0$

$\Rightarrow a = 2b = c$

Now $\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{7}{8} \Rightarrow \sin B = \frac{\sqrt{15}}{8}.$

13. Data $\Rightarrow (\cos A+\sin A)(\cos B+\sin B)=2$

$\Rightarrow \cos(A-B)+\sin(A+B)=2$

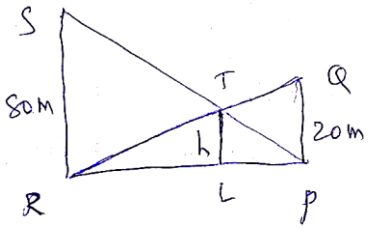
$\Rightarrow A=B,\ A+B=\frac{\pi}{2}$

$\Rightarrow A=B,\ C=\frac{\pi}{2}\Rightarrow A=B=\frac{\pi}{4}, C=\frac{\pi}{2}$

Now $\frac{a+b}{C}=\frac{\sin A+\sin B}{\sin C}=\sqrt{2}$

$\Rightarrow\left(\frac{a+b}{C}\right)=2$

14.



from figure, using similar triangles

$\frac{h}{20}=\frac{RL}{RP}, \frac{h}{80}=\frac{PL}{RP}$

Adding $\Rightarrow \frac{h}{20}+\frac{h}{80}=\frac{RL}{RP}+\frac{PL}{RP}$

$\Rightarrow \frac{5h}{80}=\frac{RL+PL}{RP}\Rightarrow \frac{h}{16}=\frac{RP}{RP}$

$\Rightarrow \frac{h}{16}=1\Rightarrow h=16$

15. Choose $z-1=\cos\theta+i\cos\theta\left(As|z-1|=1\right)\_\_\_(1)$

$(1)\Rightarrow Arg(z-1)=\theta\_\_\_\_\_(2)$

$Now(1)\Rightarrow z=1+\cos\theta+i\sin\theta$

$=2\cos^2\left(\frac{\theta}{2}\right)+2i\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

$=2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)$

$=r\cos\alpha\left(where\ \wedge=2\cos\frac{\theta}{2} and\alpha=\frac{\theta}{2}\right)$

$\Rightarrow Amp\ z=\alpha\Rightarrow Ampz=\frac{\theta}{2}$

$\Rightarrow Amp\ z=\frac{1}{2}Amp(z-1)$

16. $Data\Rightarrow x=-1+i\sqrt{2}$

$\Rightarrow x+1=i\sqrt{2}\Rightarrow (x+1)^2=-2$

$\Rightarrow x^2+2x+3=0$

Now $\Rightarrow x^4+4x^3+6x^2+4x+3=\left(x^2+2x+3\right)\left(x^2+2x-1\right)+6$

$=0+6=6$

17. Data $\Rightarrow 2z_2=z_1+z_3$

$\Rightarrow z_2 = \dfrac{z_1 + z_3}{2}$

$\Rightarrow z_2$ is mid-point of line joining the points z_1 and z_3

$\Rightarrow z_1, z_2, z_3$ are collinear.

18. Put $n = 3$ and use verification.

19. We have $\underbrace{\left|z^2 - 3\right|}_{3|z|} \geq |z|^2 - 3$

$\Rightarrow |z|^2 - 3|z| - 3 \leq 0$

$\Rightarrow 0 < |z| \leq \dfrac{3+\sqrt{21}}{2}$

20. $z = y\left(\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}\right) = 4cis\left(\dfrac{-\pi}{3}\right)$

$z^{\frac{1}{4}} = y^{\frac{1}{4}}\ cis\left(\dfrac{2k\pi}{y} - \dfrac{\pi}{12}\right)(k = 0,1,2,3)$

Now $k = 1$ gives least positive argument as $\dfrac{5\pi}{12}$

Numerical Value Questions:-

21. E=(Sum of squares of all Trigonometrical ratios)

$\Rightarrow E = \sin^2\theta + \cos^2\theta + \tan^2\theta + \cos^2\theta + \sec^2\theta + \cos ec^2\theta$

$\Rightarrow E = 3 + 2\left(\tan^2\theta + \cot^2\theta\right)$

$\Rightarrow \dfrac{E-3}{2} = \tan^2\theta + \cot^2\theta \left(\geq 2 by\ Am \geq GM\right)$

$\Rightarrow \dfrac{E-3}{2} \geq 2 \Rightarrow E \geq 7$

$\therefore$ minimum value of E is 7.

22. $a + b + c = 66, ab + bc + ca = 165, abc = 1.$

Now $\sum\left(\dfrac{\cos A}{a}\right) = \sum\left(\dfrac{b^2 + c^2 - a^2}{2abc}\right)$

$= \dfrac{1}{2}\sum\left(b^2 + c^2 - a^2\right)$

$= \dfrac{1}{2}\left(a^2 + b^2 + c^2\right)$

$= \dfrac{1}{2}\left\{(a+b+c)^2 - 2(ab+bc+ca)\right\}$

$= \dfrac{1}{2}\left\{(66)^2 - 2(165)\right\} = 2013.$

23. Simplification gives $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

$\therefore \cos\left(\tan^{-1}1+\tan^{-1}2+\tan^{-1}3\right)=\cos\pi=-1$

24. $f(x)=5x^3+Mx+N$

Also $x^2+x+1=(x-w)\left(x-w^2\right)$

Now $f(x)$ is divisible by x^2+x+1 gives

$f(w)=0,\ f\left(w^2\right)=0$

$(i.e.)5+Mw+N=0,\ 5+Mw^2+N=0$

$(i.e)M=0,\ N=-5$

Thus $M+N=-5$

25. $\bar{z}+x=0\Rightarrow\bar{z}=-z\ \_\_\_(1)$

$|z|^2-4zi=z^2\Rightarrow z\bar{z}-4zi=z^2$

$\Rightarrow\bar{z}-4i=z$

$\Rightarrow -z-4i=z\ (\because(1))$

$\Rightarrow 2\bar{z}=-4i$

$\Rightarrow z=-2i\Rightarrow|z|=2$

PHYSICS

26. Spring is divided in to length ratio 1:4

larger spring constant $k_2=5k/4$

$T\alpha\ 1/\sqrt{k}\Rightarrow\frac{T_1}{T_2}=\sqrt{\frac{k_2}{k_1}}\Rightarrow\frac{T_1}{T_2}=\sqrt{\frac{5K/4}{K}}\Rightarrow T_2=\frac{2T}{\sqrt{5}}$

27. $T\alpha\ 1/\sqrt{g}\Rightarrow\frac{\Delta T}{T}=\frac{-1}{2}\left(\frac{\Delta g}{g}\right)$

$\Delta T=\frac{-1}{2}\left(\frac{d}{R}\right)T\Rightarrow\Delta T=\frac{-1}{2}\left(\frac{1}{6400}\right)86400\Rightarrow 7$

28. $\frac{g_2}{g_1}=\left(\frac{R}{R+h}\right)^2\Rightarrow g_2=g\left(\frac{R}{R+R}\right)^2=\frac{g}{4}$

$\frac{T_2}{T_1}=\sqrt{\frac{g_1}{g_2}}\Rightarrow\frac{T_2}{T_1}=\sqrt{\frac{g}{\left(g/4\right)}}=2$

29. $T\alpha\sqrt{L} \;\; \therefore L = T, 16L = 4T$

condition for same phase $\Delta\phi = 2\pi \; \Delta\phi = \left(\frac{2\pi}{T_1} - \frac{2\pi}{T_2}\right)t$

$2\pi = \left(\frac{2\pi}{T} - \frac{2\pi}{4T}\right)t \Rightarrow t = 4/3\,T$

30. $V_{\max} = A\left(\sqrt{\frac{k}{m}}\right) \Rightarrow V_{\max} = \left(10^{-1}\right)\sqrt{\frac{200}{2}} = 1 m/s$

$\left(\therefore k = mg/x = 1\frac{(10)}{5\times10^{-2}}\right)$

31. $g_h = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{100} = \frac{R}{(R+h)^2} \Rightarrow h = 9R$

32. $g_\phi = g - R\omega^2 \Rightarrow g_\phi = g - R\left(\frac{2g}{5R}\right) = \frac{3g}{5}$

$\frac{w_2 - w_1}{w_1}\times100 = \frac{\frac{3g}{5} - g}{g}\times100 = 40\%$

33. $E_1 = \frac{-GMm}{2(2r)} + \frac{GMm}{2(r)} = \frac{GMm}{4r} \_\_\_\_\_ (1)$

$E_2 = \frac{-GMm}{2(3r)} + \frac{GMm}{2(2r)} = \frac{GMm}{12r} \_\_\_\_\_ (2)$

From (1), (2) $E_2 = E_1/3$

34. $V_e = \sqrt{2gR} \;\; \therefore \frac{V_{e1}}{V_{e2}} = \sqrt{\left(\frac{g_1}{g_2}\right)\left(\frac{R_1}{R_2}\right)}$

$\frac{V_{e1}}{V_{e2}} = \sqrt{(5)(20)} = 10$

35. $g^1 = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{64}{100} = \left(\frac{R}{R+h}\right)^2 \Rightarrow h = \frac{R}{4} = 1600\,km$

36. $\Delta\phi = \left(\frac{2\pi\eta}{V}\right)\Delta x \Rightarrow 1080\times\frac{\pi}{180} = \frac{2\pi\eta}{480}(12) \Rightarrow \eta = 120$

37. $V_1/V_2 = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{V}{\sqrt{3}V} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{T_2}{T_1} = 3$

$T_2/T_1 - 1 = 3 - 1 \Rightarrow \frac{600}{2} = T_1 \Rightarrow t_1 = 27^0 C$

38. $\lambda = 2L \Rightarrow \lambda = 2\left(5\times10^{-2}\right) = 0.1$

$V = \eta\lambda = 200(0.1) = 20\,m/s$

39. $n^1 = \frac{V}{V+V_S}n \Rightarrow \lambda^1 = \left(\frac{V+V_S}{V}\right)\lambda$

$\Rightarrow \lambda^1 = \left(1+\frac{1}{5}\right)50 = 60\,cm.$

40. Difference between two successive harmonics

$2n-n=\frac{V}{2l_o}\Rightarrow 2600-1944=\frac{328}{2l_o}\Rightarrow l_o=0.25\,m.$

41. $Aw=3{}^{w}\!/_{k}\Rightarrow k={}^{3}\!/_{A}$

$\phi=\frac{2\pi}{\lambda}x\Rightarrow\phi=kx\Rightarrow\phi=\frac{3}{A}x$

42. $n\alpha\frac{1}{\sqrt{L}}\Rightarrow\frac{\Delta n}{n}=\frac{-1}{2}\frac{(L\alpha\Delta\theta)}{L}$

$\Rightarrow\frac{1.001-1}{1}=\frac{-1}{2}\alpha(10-30)\Rightarrow\alpha=1\times10^{-4}C^{-1}$

43. $\Delta n=\left(\frac{2V_0}{V}\right)n\Rightarrow 6=\left(\frac{2V_0}{V}\right)n\_\_\_\_(1)$

$8=\left(\frac{2V_0}{V}\right)(n+100)\_\_\_\_(2)$

dividing $\frac{6}{8}=\frac{n}{n+100}\Rightarrow n=300\,Hz$

44. $n_1^1=n_2^1\Rightarrow\frac{V}{V-V_S}n_1=\frac{V}{V+V_S}n_2$

$\Rightarrow\frac{V}{V-V_S}(1)=\frac{V}{V+V_S}(2)\Rightarrow V_S=\frac{V}{3}=\frac{300}{3}=100\ m/s$

45. $V_S=r\omega=2(15)=30\,m/s$

$n_{\max}=\frac{V}{V-V_S}N\Rightarrow n_{\max}=\left(\frac{330}{330-30}\right)540=594\,Hz$

46. $\frac{a_{\max}}{V_{\max}}=\frac{\omega^2A}{\omega A}=\frac{48}{12}$

$\Rightarrow\omega=4\,rad/s\ \ \therefore A=\frac{V_{\max}}{\omega}=\frac{12}{4}=3m$

47. $\frac{T_1^2}{T_2^2}=\left(\frac{R_1}{R_2}\right)^3\Rightarrow\frac{24}{T_2}=\left(\frac{6400}{36000}\right)^{3/2}$

$\Rightarrow T_2=1hour$

48. $\frac{V_1}{V_2}=\sqrt{\frac{R+h_2}{R+h_1}}=\sqrt{\frac{6400+49800}{6400+6400}}=\frac{2}{1}$

49. $\frac{n_1}{n_2}=\sqrt{\frac{T_1}{T_2}}\Rightarrow\frac{V/4l}{3V/4l}=\sqrt{\frac{T}{T+8}}\Rightarrow T=1N$

50. $n\alpha\sqrt{T}\Rightarrow\frac{\Delta n}{n}=\frac{1}{2}.\frac{\Delta T}{T}\Rightarrow\frac{\Delta n}{300}=\frac{1}{2}\frac{4}{300}$

$\Rightarrow\Delta n=2\,beats/\sec.$