



MATHS

SYLLABUS: Dot product, Cross product, Product of three vectors, Product of four vectors

- If $\vec{a} = x\vec{i} + (x-1)\vec{j} + \vec{k}$ and $\vec{b} = (x+1)\vec{i} + \vec{j} + a\vec{k}$ make acute angle for all $x \in R$ then $a \in$

1) (0,1) 2) (1,∞) 3) (2,∞) 4) (-∞,2)
- If $\vec{a} + 2\vec{b} + 3\vec{c} = 4$ then the least value of $a^2 + b^2 + c^2$ is

1) $\frac{6}{7}$ 2) $\frac{8}{7}$ 3) $\frac{10}{7}$ 4) $\frac{12}{7}$
- The unit vector \vec{a} which makes an angle $\frac{\pi}{4}$ with z axis and it is such that $\vec{a} + \vec{i} + \vec{j}$ a unit vector, is

1) $-\frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$ 2) $-\frac{1}{2}\vec{i} - \frac{1}{2}\vec{j} - \frac{1}{\sqrt{2}}\vec{k}$ 3) $\frac{1}{2}\vec{i} - \frac{1}{2}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$ 4) $-\frac{1}{2}\vec{i} - \frac{1}{2}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$
- If the vector $3\vec{p} + \vec{q}; 5\vec{p} - 3\vec{q}$ and $2\vec{p} + \vec{q}; 4\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors then the angle between \vec{p} and \vec{q} is

1) $\cos^{-1} \frac{3}{8}$ 2) $\cos^{-1} \frac{5}{8}$ 3) $\cos^{-1} \left(\frac{7}{8} \right)$ 4) $\cos^{-1} \frac{1}{8}$
- Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors, such that $\vec{a} + \vec{b} + \vec{c} = x, \vec{a} \cdot x = 1, \vec{b} \cdot x = \frac{3}{2}, 1 \cdot x = 2$ then angle between \vec{c} and x is

1) $\cos^{-1} \frac{1}{4}$ 2) $\cos^{-1} \frac{1}{2}$ 3) $\cos^{-1} \left(\frac{3}{4} \right)$ 4) $\cos^{-1} \frac{2}{3}$
- If \vec{a} and \vec{b} are unit vectors, then the greatest value of $|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is

1) $\sqrt{2}$ 2) $2\sqrt{2}$ 3) $3\sqrt{2}$ 4) $4\sqrt{2}$
- Let \vec{a}, \vec{b} and \vec{c} be three vectors, such that $\vec{a} \neq \vec{o}, |\vec{a}| = |\vec{c}| = 1, |\vec{b}| = 4$ and $|\vec{b} \times \vec{c}| = \sqrt{15}$.

If $\vec{b} - 2\vec{c} = \lambda\vec{a}$ then the value of λ is

1) ± 1 2) ± 2 3) ± 3 4) ± 4
- The position vectors of the vertices of quadrilateral is

$\frac{1}{2}lm|(\vec{b} \times \vec{d})|$ 2) $\frac{1}{2}(l+m)|\vec{b} \times \vec{d}|$ 3) $\frac{l}{2m}|\vec{b} \times \vec{d}|$ 4) $\frac{1}{4}(l+m)|\vec{b} \times \vec{d}|$
- Let \vec{a}, \vec{b} be unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$ then the value of $(2\vec{a} + 5\vec{b}) \cdot (3\vec{a} + \vec{b} + \vec{a} \times \vec{b})$ is

1) $\frac{39}{2}$ 2) $\frac{39}{4}$ 3) $\frac{39}{8}$ 4) $\frac{39}{11}$
- If the vectors $\vec{c}, \vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{b} = \vec{j}$ are such that $\vec{a}, \vec{c}, \vec{b}$ form a right handed system then $\vec{c} =$

1) $x\vec{i} - y\vec{j}$ 2) $-x\vec{i} - y\vec{j}$ 3) $x\vec{k} - z\vec{i}$ 4) $z\vec{i} - x\vec{k}$

11. If \vec{a}, \vec{b} and \vec{c} be three non-zero vectors such that $\lambda(\vec{b} \times \vec{a}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = \vec{0}$ then the value of λ is
 1) -1 2) 0 3) 1 4) 2
12. Let $\vec{a} = x\vec{i} + 12\vec{j} - \vec{k}, \vec{b} = 2\vec{i} + 2x\vec{j} + \vec{k}, \vec{c} = \vec{i} + \vec{k}$. If $\vec{a}, \vec{b}, \vec{c}$ form left handed system then $x \in$
 1) (-1, -2) 2) (2, 3) 3) (-3, 2) 4) (-3, -2)
13. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors and $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$. If the angle between \vec{b} and \vec{c} is $\frac{\pi}{3}$ then the value of $|\vec{a}, \vec{b}, \vec{c}|$ is
 1) $\frac{\sqrt{3}}{2}$ 2) $\frac{\sqrt{3}}{4}$ 3) $\frac{\sqrt{3}}{2\sqrt{2}}$ 4) $\frac{\sqrt{3}}{4\sqrt{2}}$
14. The altitude of a parallelepiped whose three continuous are vectors $\vec{a} = \vec{i} + \vec{j} + \vec{k}, \vec{b} = 2\vec{i} + 4\vec{j} - \vec{k}, \vec{c} = \vec{i} + \vec{j} + 3\vec{k}$ with \vec{a} and \vec{b} as sides of the base of the parallelepiped, is
 1) $\frac{\sqrt{38}}{19}$ 2) $\frac{2\sqrt{38}}{19}$ 3) $\frac{3\sqrt{38}}{19}$ 4) $\frac{4\sqrt{38}}{19}$
15. If $\vec{i} \times [(\vec{a} - \vec{j}) \times \vec{i}] + \vec{j} \times [(\vec{a} - \vec{k}) \times \vec{j}] + \vec{k} \times [(\vec{a} - \vec{i}) \times \vec{k}] = \vec{0}$ then $\vec{a} =$
 1) $\frac{\vec{i} + \vec{j} + \vec{k}}{4}$ 2) $\frac{\vec{i} + \vec{j} + \vec{k}}{3}$ 3) $\frac{\vec{i} + \vec{j} + \vec{k}}{2}$ 4) $\vec{i} + \vec{j} + \vec{k}$
16. If $\vec{\alpha} \parallel (\vec{\beta} \times \vec{r})$ then $(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{r})$ equals to
 1) $|\alpha|^2 (\vec{\beta} \cdot \vec{r})$ 2) $|\beta|^2 (\vec{r} \cdot \vec{\alpha})$ 3) $|\vec{r}|^2 (\vec{\alpha} \cdot \vec{\beta})$ 4) $|\vec{\alpha}| |\vec{\beta}| |\vec{r}|$
17. Let $\vec{f}(t) = [t]\vec{i} + (t - [t])\vec{j} + (t + 1)\vec{k}$, where $[\cdot]$ denotes the greatest integer function then the vectors $\vec{f}\left(\frac{5}{4}\right)$ and $\vec{f}(t), 0 < 1 < 2$
 1) parallel to each other 2) perpendicular to each other
 3) inclined at $\cos^{-1}\left(\frac{2}{\sqrt{7}(1-t^2)}\right)$ 4) inclined at $\cos^{-1}\left(\frac{8+t}{9\sqrt{1+t^2}}\right)$
18. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 0$ then $(\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$ is equal to
 1) $48 \frac{\wedge}{a}$ 2) $-48 \frac{\wedge}{b}$ 3) $48 \frac{\wedge}{a}$ 4) $-48 \frac{\wedge}{a}$
19. If $4\vec{a} + 5\vec{b} + 9\vec{c} = \vec{0}$ then $(\vec{a} \times \vec{b}) \times [(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})]$ is equal to
 1) a vector perpendicular to the plane of $\vec{a}, \vec{b}, \vec{c}$ 2) unit vector
 3) $\vec{0}$ 4) a scalar
20. If $\vec{a}, \vec{b}, \vec{c}$ are any three non-coplanar vectors then the equation $[\vec{b} \times \vec{c} \quad \vec{c} \times \vec{a} \quad \vec{a} \times \vec{b}] x^2 + [\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] x + [\vec{b} - \vec{c} \quad \vec{c} - \vec{a} \quad \vec{a} - \vec{b}] = 0$ has roots
 1) 1 2) 2 3) 3 4) 4

Syllabus:

- If $f(x) = a \log|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$ then
 - $a = 2, b = \frac{1}{2}$
 - $a = \frac{1}{2}, b = 2$
 - $a = 2, b = \frac{-1}{2}$
 - $a = -2, b = \frac{-1}{2}$
- Let $p(x) = a_0 + a_1x^2 + a_2x^4 + \dots + a_nx^{2n}$ be a polynomial in real variable x with $0 < a_0 < a_1 < a_2 < \dots < a_n$ the function $p(x)$ has
 - neither maximum nor minimum
 - only one maximum
 - only one minimum
 - only one maximum and only one minimum
- On the interval $[0,1]$, the function $x^{25}(1-x)^{75}$ takes its maximum value at the point
 - 0
 - $\frac{1}{4}$
 - $\frac{1}{2}$
 - $\frac{1}{3}$
- The minimum value of $f(x) = x^8 + x^6 - x^4 - 2x^3 - x^2 - 2x + 9$ on the real number set \mathbf{R} is
 - 1
 - 5
 - 0
 - 3
- The minimum value of $z = 2x^2 + 2xy + y^2 - 2x + 2y + 2$ is
 - 2
 - 2
 - 3
 - 3
- The value of a such that the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assumes least value is
 - 0
 - 1
 - 1
 - 2
- $f(x) = 2 - (x-1)^{\frac{2}{3}}$ is maximum at
 - $x = -1$
 - $x = 1$
 - $x = 0$
 - $x = 2\sqrt{2}$
- If $p(x)$ is a polynomial of degree 3 satisfying $p(-1) = 10, p(1) = -6, p'(x)$ has local minima at $x = 1$ and $p(x)$ has maximum at $x = -1$ then the distance between local maxima and local minima of the curve is
 - $2\sqrt{65}$
 - $3\sqrt{65}$
 - $4\sqrt{65}$
 - $5\sqrt{65}$
- The sum of an infinitely decreasing geometric progression is equal to the least value of the function $f(x) = 3x^2 - x + \frac{25}{12}$ and first term of the progression is equal to the square of its common ratio then the common ratio is
 - $\sqrt{2} - 1$
 - $\sqrt{3} - 1$
 - $2 - \sqrt{3}$
 - $\frac{2\sqrt{2}}{3}$
- For $x \geq 0$, the least value of the expression $\frac{1+x^2}{1+x}$ is
 - $\sqrt{2}$
 - 1
 - $2(\sqrt{2} + 1)$
 - $2(\sqrt{2} - 1)$
- The maximum value of $f(x) = \cos x \cdot \sqrt{\sin x}$ on the interval $\left[0, \frac{\pi}{2}\right]$ is
 - $2^{\frac{1}{2}} \cdot 3^{\frac{-3}{4}}$
 - $2^{\frac{1}{2}} \cdot 3^{\frac{3}{4}}$
 - $2^{\frac{3}{4}} \cdot 3^{\frac{1}{2}}$
 - $2^{\frac{3}{4}} \cdot 3^{\frac{-1}{2}}$
- The function $f(x) = x^3 - 3x + 3$ on the interval $\left[-3, \frac{3}{2}\right]$ let $M = \max f(x)$ and $m = \min f(x)$ on $\left[-3, \frac{3}{2}\right]$ then
 - $M=15, m=5$
 - $M=5, m=15$
 - $M=15, m=-5$
 - $M=-5, m=-15$
- The function $f(x) = x + \tan x$ has

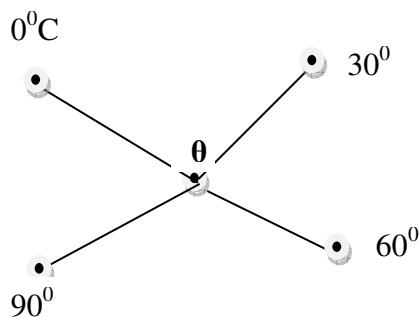
- 1) only one maximum and one minimum 2) only one maximum
3) only one minimum 4) neither maximum nor minimum
14. The number of critical points for the function $f(x) = \frac{1}{3} \sin \theta \cdot \tan^2 x + (\sin \theta - 1) \tan x + \sqrt{\frac{\theta - 2}{8 - \theta}}$ where $\pi < \theta < 2\pi$ is
1) 4 2) 2 3) 1 4) 0
15. A closed circular cylinder of volume K cubic units is to be formed with minimum amount of material the ratio of its height to the radius of base is
1) 4 2) 3 3) 2 4) 3/2
16. From a rectangular cardboard of size 3x8, equal square pieces are removed from the four corners and an open rectangular box is formed from the remaining. The maximum volume of the box is
1) $\frac{250}{6}$ cubic units 2) $\frac{250}{3}$ cubic units 3) 125 cubic units 4) $\frac{200}{27}$ cubic units
17. Let x, y, z be positive numbers such that $x + y + z = 26$ and $y = 3x$. If $x^2 + y^2 + z^2$ is to be least, then the value of x is
1) 3 2) 4 3) 3.5 4) 4.5
18. If h is the height of a circular cone of greatest volume of given slant height l then h is equal to
1) $\frac{l}{\sqrt{3}}$ 2) $\frac{l}{\sqrt{2}}$ 3) $\sqrt{3}l$ 4) $\sqrt{2}l$
19. The least distance of the point $Q(0, -2)$ from the point $p(x, y)$ where $y = \frac{16}{\sqrt{3}x^3} - 2$ and $x > 0$
1) $\frac{4}{3}$ 2) $4\sqrt{3}$ 3) $\frac{4}{\sqrt{3}}$ 4) $3\sqrt{3}$
20. The greatest volume of a circular cylinder whose total surface area is 2π is
1) $\frac{2\pi}{3\sqrt{3}}$ 2) $\frac{2\pi}{3}$ 3) $\frac{2\pi}{\sqrt{3}}$ 4) $\frac{4\pi}{3\sqrt{3}}$

PHYSICS

Syllabus : Heat Transfer & Kinetic Theory Of Gases

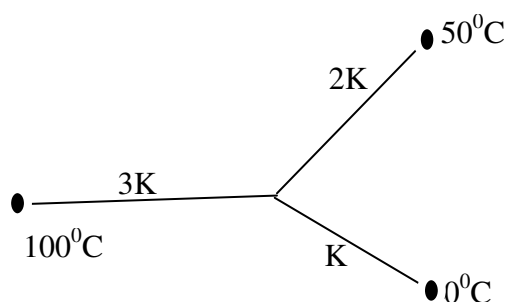
- Four molecules of gas have speeds 1, 2, 3 and 4 km/s. The value of the r.m.s. speed of the gas molecules is
1) $\frac{1}{2}\sqrt{15} \text{ km/s}$ 2) $\frac{1}{2}\sqrt{10} \text{ km/s}$ 3) 2.5 km/s 4) $\sqrt{\frac{15}{2}} \text{ km/s}$
- A mixture of ideal gases has 2 moles of He, 4 moles of oxygen and 1 mole of ozone at absolute temperature T. The internal energy of mixture is?
1) 13 RT 2) 11 RT 3) 16 RT 4) 14 RT
- When one mole of monatomic gas is mixed with one mole of a diatomic gas, then the equivalent value of γ for the mixture will be (vibrational mode neglected)
1) 1.33 2) 1.40 3) 1.90 4) 1.6
- By increasing temperature of a gas by 6°C its pressure increased by 0.4% at constant volume. Then initial temperature of gas is?
1) 1000 K 2) 1500 K 3) 2000 K 4) 750 K
- The raise in the temperature of a given mass of an ideal gas at constant pressure and at temperature 27°C to double its volume is?
1) 327°C 2) 54°C 3) 300°C 4) 600°C

6. The average velocity of gas molecules is
- 1) proportional to \sqrt{T}
 - 2) proportional to T
 - 3) Zero
 - 4) Not possible to determine
7. The internal energy of 10g of nitrogen at N.T.P. is about
- 1) 2575 J
 - 2) 2025 J
 - 3) 3721 J
 - 4) 4051 J
8. The equation of state corresponding to 8g of O_2 is ?
- 1) $PV = 8RT$
 - 2) $PV = \frac{RT}{4}$
 - 3) $PV = RT$
 - 4) $PV = \frac{RT}{2}$
9. The temperature of gas is raised from $27^\circ C$ to $927^\circ C$. The r.m.s. speed is?
- 1) $\sqrt{\frac{927}{27}}$ times the earlier value
 - 2) Remains same
 - 3) Gets halved
 - 4) Get doubled
10. Two thermally insulated vessels, 1 and 2 are filled with air at temperatures (T_1, T_2) , volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessel is opened, the temperature inside the vessel at equilibrium will be
- 1) (T_1, T_2)
 - 2) $\frac{T_1 + T_2}{2}$
 - 3) $\frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$
 - 4) $\frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_1 + P_2 V_2 T_2}$
11. What is the dimensional formula for thermal resistance
- 1) $[M^{-1} L^{-1} T^{-1} K]$
 - 2) $[ML^2 T^{-2} K^{-1}]$
 - 3) $[ML^{-3} T^2 K^{-1}]$
 - 4) $[M^{-1} L^{-2} T^3 K]$
12. Four rods of same material with different radii r and length l are used to connect two reservoirs of heat at different temperatures which one will conduct maximum heat?
- 1) $r=1$ cm, $l=1$ m
 - 2) $r=2$ cm, $l=2$ m
 - 3) $r=1$ cm, $l=\frac{1}{2}$ m
 - 4) $r=2$ cm, $l=\frac{1}{2}$ m
13. Four rods of same material and having the same cross section and length have been joined as shown. The temperature of the junction of four rods will be



- 1) $20^\circ C$
 - 2) $30^\circ C$
 - 3) $45^\circ C$
 - 4) $60^\circ C$
14. A uniform thermometer scale at steady state with its 0 cm mark at $20^\circ C$ and 100 cm mark at $100^\circ C$ temperature of the 60 cm mark is
- 1) $48^\circ C$
 - 2) $68^\circ C$
 - 3) $52^\circ C$
 - 4) $58^\circ C$
15. A good absorber is a good emitter is explained by
- 1) Stefan's law
 - 2) Wein's law
 - 3) Newton's law of cooling
 - 4) Kirchoff's law

16. Three rods of same dimensions have thermal conductivities $3K, 2K$ and K . They are arranged as shown with their ends at 100°C , 50°C and 0°C . The temperature of their junction is



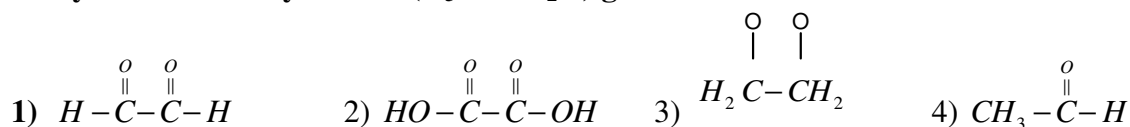
- 1) 75°C 2) $\frac{2}{3}^{\circ}\text{C}$ 3) 40°C 4) $\frac{100}{3}^{\circ}\text{C}$
17. If wave length of maximum intensity of radiation emitted by sun and moon are $0.5 \times 10^{-6}\text{m}$ and 10^{-4}m respectively, then the ratio of their temperature is
- 1) $\frac{1}{10}$ 2) $\frac{1}{50}$ 3) 100 4) 200
18. If transmission power of surface is $\frac{1}{9}$, reflective power is $\frac{1}{6}$, then what is its absorptive power?
- 1) $\frac{18}{13}$ 2) $\frac{13}{18}$ 3) $\frac{3}{15}$ 4) $\frac{15}{3}$
19. A black body is at 727°C . It emits energy at a rate which is proportional to
- 1) $(1000)^4$ 2) $(1000)^2$ 3) $(727)^4$ 4) $(727)^2$
20. A beaker full of hot water is kept in a room. If it cools from 80°C to 75°C in t_1 minutes, from 75°C to 70°C in t_2 minutes and from 70°C to 65°C in t_3 minutes, then
- 1) $t_1 < t_2 < t_3$ 2) $t_1 > t_2 > t_3$ 3) $t_1 = t_2 = t_3$ 4) $t_1 < t_2 = t_3$

CHEMISTRY

Syllabus: ALKYNES, BENZENE

- Gem dihalides on treatment with alcoholic KOH followed by NaNH_2 gives
 - Alkyne
 - Alkene
 - Alkane
 - cycloalkane
- 1-Pentyne and 2-Pentyne can be distinguished by
 - Silver mirror test
 - Iodoform test
 - Addition of H_2
 - Baeyer's test
- The reagent used for obtaining trans-alkene from alkyl substituted acetylene with hydrogen is
 - LiAlH_4
 - $\text{Zn} + \text{HCl}$
 - Na in liquid NH_3
 - H_2 in presence of Ni
- $\text{H} - \text{C} \equiv \text{C} - \text{H} + 2\text{Na} \rightarrow \text{A} \xrightarrow[\text{CH}_3\text{Cl}]{2 \text{ moles of}} \text{B}$; then B is
 - 1-Butyne
 - 2-Butyne
 - 2-Pentyne
 - 3-Pentanone
- Alkynes exhibit functional isomerism with
 - Alkanes
 - Alkadienes
 - Alcohols
 - Alkenes
- Acetylene on treatment with H_2O in the presence of HgSO_4 and H_2SO_4 gives
 - Ethane
 - Ethanal
 - Ethanol
 - Ethanoic acid
- $\text{X} + 2\text{Zn} \xrightarrow{\text{alcohol}} \text{H} - \text{C} \equiv \text{C} - \text{H}$; Here 'X' is
 - 1,1 Dibromomethane
 - 1,2 Dibromomethane
 - Dibromo ethane
 - 1,1,2,2-tetrabromoethane

8. Acetylene on ozonolysis with (O_3+Zn/H_2O) gives



9. The reduction of 4-octyne with H_2 in the presence of $Pd/BaSO_4$ in quinoline gives

- 1) Trans-4-Octene 2) cis-4-octene
3) A mixture of Cis and trans -4-Octene 4) A completely reduced product C_8H_{18}

10. The C-C bond length is shortest in

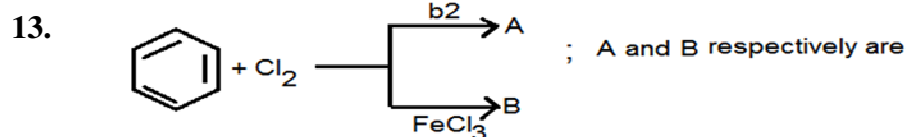
- 1) C_2H_6 2) C_6H_6 3) C_2H_2 4) C_2H_4

11. Preparation of benzene from Phenol is

- 1) Reduction 2) Oxidation 3) Addition 4) Dehydration

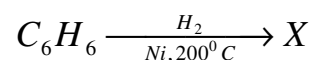
12. Benzene reacts with ----- to yield acetophenone

- 1) $C_2H_5COCl + AlCl_3$ 2) $C_2H_5COCl / AlCl_3$ 3) $CH_3COCl / AlCl_3$ 4) $CH_3Cl / AlCl_3$



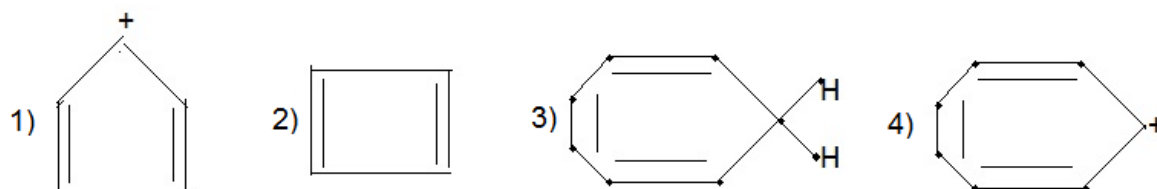
- 1) Hexachlorocyclohexane and C_6H_5Cl
2) Chlorobenzene, Hexachlorocyclohexane
3) o- and p- Dichlorobenzene
4) chlorobenzene and $C_6H_5Cl_4$

14. Number of σ $sp^2 - sp^2$ bonds present in a molecule of 'X' in the process is



- 1) 6 2) 3 3) 12 4) Zero

15. Which of the following compound / ion is expected to be aromatic



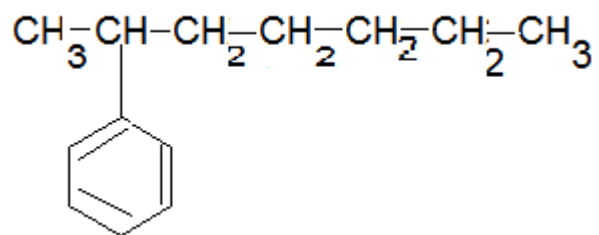
16. $-COOH$ group in electrophilic substitution directs the incoming group to

- 1) O-position 2) p-position
3) m-position 4) o- & p-positions

17. Benzene contains double bonds but it does not addition reactions readily because

- 1) Double bonds in benzene ring are strong
2) Double bonds change their position rapidly
3) Resonance lowers the energy of benzene molecules and leads to greater stabilization.
4) Benzene has cumulative double bonds.

18. IUPAC Name of the following compound is



- 1) Heptyl benzene
2) 2-Benzyle heptane
3) 2-Phenyl heptane
4) 1-Heptyl benzene
19. **The dipole moment of benzene is**
1) Zero
2) less than p-dichlorobenzene
3) Greater than p-dichlorobenzene
4) equal to that of chlorobenzene
20. **In nitrating mixture, HNO₃ acts as a**
1) Base
2) Acid
3) Reducing Acid
4) Catalyst



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SR EAMCET
Time: 3 Hours

DPP -16

Date: 26-04-2020

KEY SHEET

MATHS-A

1) 3	2) 2	3) 4	4) 1	5) 3	6) 2	7) 4	8) 2	9) 1	10) 4
11) 4	12) 3	13) 1	14) 2	15) 3	16) 1	17) 4	18) 1	19) 3	20) 3

MATHS-B

1) 3	2) 3	3) 2	4) 2	5) 4	6) 3	7) 2	8) 3	9) 2	10) 4
11) 1	12) 2	13) 4	14) 4	15) 3	16) 4	17) 2	18) 1	19) 3	20) 1

PHYSICS

1) 4	2) 3	3) 3	4) 2	5) 1	6) 3	7) 2	8) 2	9) 4	10) 3
11) 4	12) 4	13) 3	14) 2	15) 4	16) 2	17) 4	18) 2	19) 1	20) 1

CHEMISTRY

1) 1	2) 1	3) 3	4) 2	5) 2	6) 2	7) 4	8) 1	9) 2	10) 3
11) 1	12) 3	13) 1	14) 4	15) 4	16) 3	17) 3	18) 3	19) 1	20) 1

HINTS & SOLUTIONS
MATHS-A

1. Sol:

$$\begin{aligned}\bar{a}\bar{b} &= x(x+1) + (x-1) + a \\ &= x^2 + 2x + (a-1)\end{aligned}$$

$$\bar{a}\bar{b} > 0 \forall x \in R \Rightarrow x^2 + 2x + a - 1 > 0$$

$$\forall x \in R \Rightarrow \Delta < 0 \Rightarrow 4 - 4(1)(a-1) < 0$$

$$\Rightarrow 1 - a + 1 < 0 \Rightarrow a > 2 \Rightarrow a \in (2, \infty)$$

2. Sol:

$$\text{Let } \bar{p} = a\bar{i} + b\bar{j} + c\bar{k}$$

$$\bar{q} = \bar{i} + 2\bar{j} + 3\bar{k}, \theta \in (\bar{p}, \bar{q})$$

$$\cos \theta = \frac{a + 2b + 3c}{\sqrt{a^2 + b^2 + c^2} \sqrt{1 + 4 + 9}}$$

$$\cos^2 \theta = \frac{16}{(a^2 + b^2 + c^2)(14)} \because a + 2b + 3c = 4$$

$$\cos^2 \theta \leq 1$$

$$\frac{8}{7(a^2 + b^2 + c^2)} \leq 1 \Rightarrow a^2 + b^2 + c^2 \leq \frac{\infty}{7}$$

$$\text{least value of } a^2 + b^2 + c^2 \text{ is } \frac{8}{7}$$

3. Sol:

$$\text{Let } \bar{a} = x\bar{i} + y\bar{j} + z\bar{k}, |\bar{a}| = 1$$

$$\Rightarrow x^2 + y^2 + z^2 = 1, (\bar{a}, \bar{k}) = \frac{\pi}{4},$$

$$\cos \frac{\pi}{4} = \frac{\bar{a} \cdot \bar{k}}{|\bar{a}| |\bar{k}|} \Rightarrow z = \frac{1}{\sqrt{2}}$$

$$|\bar{a} + \bar{i} + \bar{j}| = 1 \Rightarrow \sqrt{(x+1)^2 + (y+1)^2 + z^2} = 1$$

$$\Rightarrow x^2 + y^2 + z^2 + 2x + 2y + 1 = 0$$

$$\Rightarrow 1 + 2x + 2y + 1 = 0$$

$$\Rightarrow y = -(x+1)$$

$$x^2 + (x+1)^2 + \frac{1}{2} = 1$$

$$2x^2 + 2x + X + \frac{1}{2} = X$$

$$\Rightarrow 4x^2 + 4x + 1 = 0$$

$$\Rightarrow (2x+1)^2 = 0 \Rightarrow x = -\frac{1}{2}$$

$$\Rightarrow y = -\frac{1}{2}$$

$$\Rightarrow \bar{a} = -\frac{1}{2}\bar{i} - \frac{1}{2}\bar{j} + \frac{1}{\sqrt{2}}\bar{k}$$

4. Sol:

$3\bar{p} + \bar{q}$ and $4\bar{p} - 2\bar{q}$ are perpendicular

$$\Rightarrow (3\bar{p} + \bar{q}) \cdot (4\bar{p} - 2\bar{q}) = 0$$

$$\Rightarrow 5|\bar{p}|^2 - 3|\bar{q}|^2 = 4\bar{p} \cdot \bar{q} \rightarrow 1$$

$2\bar{p} + \bar{q}$ and $4\bar{p} - 2\bar{q}$ are \perp

$$\Rightarrow (2\bar{p} + \bar{q}) \cdot (4\bar{p} - 2\bar{q}) = 0$$

$$\Rightarrow 8|\bar{p}|^2 - 2|\bar{q}|^2 = 0 \Rightarrow |\bar{q}|^2 = 4|\bar{p}|^2 \rightarrow 2$$

$$1 + 2 \Rightarrow 15|\bar{p}|^2 - 12|\bar{p}|^2 = 4\bar{p} \cdot \bar{q}$$

$$\Rightarrow 3|\bar{p}|^2 = 4\bar{p} \cdot \bar{q} \rightarrow 3$$

$$\cos \theta = \frac{\bar{p} \cdot \bar{q}}{|\bar{p}| \cdot |\bar{q}|} = \frac{\frac{3}{4}|\bar{p}|^2}{|\bar{p}|(2|\bar{p}|)} = \frac{3}{8}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{3}{8}\right)$$

5. Sol:

$$\bar{a} + \bar{b} + \bar{c} = x \cdot \bar{a} + x \cdot \bar{b} + x \cdot \bar{c} = x \cdot x$$

$$\Rightarrow 1 + \frac{3}{2} + x \cdot \bar{c} = 4 \Rightarrow x \cdot \bar{c} = \frac{3}{2}$$

$$\theta = (\bar{c}, x) \Rightarrow \cos \theta = \frac{\bar{c} \cdot x}{|\bar{c}| |x|}$$

$$\Rightarrow \cos \theta = \frac{\frac{3}{2}}{(1)(2)} = \frac{3}{4} \Rightarrow \theta = \cos^{-1} \frac{3}{4}$$

6. Sol:

$$\text{Let } (\bar{a}, \bar{b}) = \theta$$

$$\begin{aligned} |\bar{a} + \bar{b}|^2 &= |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a} \cdot \bar{b} \\ &= 1 + 1 + 2|\bar{a}||\bar{b}|\cos\theta \\ &= 2 + 2\cos\theta = 2\left(2\cos^2\frac{\theta}{2}\right) \end{aligned}$$

$$|\bar{a} + \bar{b}| = 2\cos\frac{\theta}{2}$$

$$\text{similarly } |\bar{a} - \bar{b}| = 2\sin\frac{\theta}{2}$$

$$\Rightarrow |\bar{a} + \bar{b}| + |\bar{a} - \bar{b}| = 2\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2}$$

$$\max = c + \sqrt{a^2 + b^2} = 0 + \sqrt{4 + 4} = 2\sqrt{2}$$

7. sol:

$$\text{Let } (\bar{b}, \bar{c}) = \theta$$

$$|\bar{b} \times \bar{c}| = \sqrt{15} \Rightarrow |\bar{b}||\bar{c}|\sin\theta = \sqrt{15}$$

$$4\sin\theta = \sqrt{15} \Rightarrow \sin\theta = \frac{\sqrt{15}}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{4} \text{ Now } \bar{b} - 2\bar{c} = \lambda\bar{a}$$

$$\Rightarrow |\bar{b} - 2\bar{c}|^2 = \lambda^2 1 = 1^2$$

$$\Rightarrow |\bar{b}|^2 + 4|\bar{c}|^2 - 4\bar{b} \cdot \bar{c} = \lambda^2$$

$$\Rightarrow 16 + 4 - 4\bar{b} \cdot \bar{c} = \lambda^2$$

$$\Rightarrow 20 - 4|\bar{b}||\bar{c}|\cos\theta = \lambda^2$$

$$\Rightarrow 20 - 4 \times 4 \times 1 \times \frac{1}{4} = \lambda^2$$

$$\Rightarrow \lambda^2 = 16 \Rightarrow \lambda = \pm 4$$

8. Sol:

Area of quadrilateral ABCD

$$= \frac{1}{2} |\overrightarrow{AC} \times \overrightarrow{BD}| = \frac{1}{2} |(l\bar{b} + m\bar{d}) \times (\bar{d} - \bar{b})|$$

$$= \frac{1}{2} |l(\bar{b} \times \bar{d}) - \bar{o} + m(\bar{o}) - m(\bar{d} \times \bar{b})|$$

$$= \frac{1}{2} |l(\bar{b} \times \bar{d}) + m(\bar{b} \times \bar{d})|$$

$$= \frac{1}{2} (l + m)(\bar{b} \times \bar{d})$$

9. Sol:

$$|\bar{a} + \bar{b}|^2 = 3 \Rightarrow |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a} \cdot \bar{b} = 3$$

$$\bar{a} \cdot \bar{b} = \frac{1}{2}$$

$$(2\bar{a} + 5\bar{b}) \cdot (3\bar{a} + \bar{b} + \bar{a} \times \bar{b})$$

$$= 6 + 2\bar{a} \cdot \bar{b} + 15\bar{b} \cdot \bar{a} + 5$$

$$= 11 + 17\bar{a} \cdot \bar{b} = 12 + 17 \times \frac{1}{2} = \frac{39}{2}$$

10. Sol:

Since $\bar{a}, \bar{c}, \bar{b}$ form a right handed system

$$\Rightarrow \bar{c} = \bar{b} \times \bar{a}$$

$$\Rightarrow \bar{j} \times (x\bar{i} + y\bar{j} + z\bar{k})$$

$$\Rightarrow -x\bar{k} + z\bar{i} = z\bar{i} - x\bar{k}$$

11. Sol:

$\bar{a} + \bar{b} + \bar{c} = \bar{0} \Rightarrow \bar{a}, \bar{b}, \bar{c}$ represent the sides of a triangle

$$\exists = x\bar{b} = \bar{b} \times \bar{c} = \bar{c} \times \bar{a}$$

$$\text{Now } \bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a} = 3(\bar{a} \times \bar{b})$$

$$\lambda(\bar{b} \times \bar{a}) + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}$$

$$\lambda(\bar{b} \times \bar{a}) + \bar{a} \times \bar{b} + \bar{a} \times \bar{b}$$

$$\lambda(\bar{b} \times \bar{a}) - 2(\bar{b} \times \bar{a}) = \bar{0}$$

$$\Rightarrow \lambda = 2$$

12. Sol:

For a left handed system

$$[\bar{a} \ \bar{b} \ \bar{c}] < 0$$

$$\Rightarrow \begin{vmatrix} x & 12 & -1 \\ 2 & 2x & 1 \\ 1 & 0 & 1 \end{vmatrix} < 0$$

$$\Rightarrow x(2x) - 12(2-1) - 1(-2x) < 0$$

$$\Rightarrow 2x^2 - 12 + 2x < 0$$

$$\Rightarrow x^2 + x - 6 < 0 \exists (x+3)(x-2) < 0$$

$$\Rightarrow x \in (-3, 2)$$

13. Sol:

$\bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{c} = 0 \exists \bar{a}$ is perpendicular to vectors

$$\bar{b} \text{ and } \bar{c} \Rightarrow \bar{a} = \lambda(\bar{b} \times \bar{c})$$

$$\Rightarrow |\bar{a}| = |\lambda| |\bar{b} \times \bar{c}| \Rightarrow 1 = |\lambda| \frac{\sqrt{3}}{2} = 1$$

$$\Rightarrow |\lambda| = \frac{2}{\sqrt{3}} |(\bar{a} \cdot \bar{b} \cdot \bar{c})| = |\bar{a} \cdot (\bar{b} \times \bar{c})|$$

$$\Rightarrow |\lambda(\bar{b} \times \bar{c}) \cdot (\bar{b} \times \bar{c})| = |\lambda| |\bar{b} \times \bar{c}|^2$$

$$\Rightarrow \frac{2}{\sqrt{3}} \times (1)(1) \left(\frac{\sqrt{3}}{2} \right)^2 = \frac{\sqrt{3}}{2}$$

14. Sol:

$$n = \frac{\text{volume of parallelepiped}}{\text{Area of base}}$$

$$= \frac{[\bar{a} \ \bar{b} \ \bar{c}]}{|\bar{a} \times \bar{b}|}$$

$$= \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix}}{\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 4 & -1 \end{vmatrix}}$$

$$= \frac{4\sqrt{38}}{38} = \frac{2\sqrt{38}}{19}$$

15. Sol:

$$\bar{i} \times [(\bar{a} - \bar{j}) \times \bar{i}]$$

$$= [\bar{i} \cdot \bar{i}](\bar{a} - \bar{j}) - (\bar{i} \cdot (\bar{a} - \bar{j}))\bar{i}$$

$$= \bar{a} - \bar{j} - (\bar{a} \cdot \bar{i})\bar{i}$$

$$\text{Similarly } \bar{j} \times [(\bar{a} - \bar{k}) \times \bar{j}]$$

$$= \bar{a} - \bar{k} + (\bar{a} \cdot \bar{j})\bar{j}$$

$$\bar{k} \times [(\bar{a} \cdot \bar{i}) \times \bar{k}] = \bar{a} - \bar{i} + (\bar{a} \cdot \bar{k})\bar{k}$$

$$3\bar{a} - (\bar{i} + \bar{j} + \bar{k}) - \bar{a} = \bar{0}$$

$$\Rightarrow \bar{a} = \frac{1}{2}(\bar{i} + \bar{j} + \bar{k})$$

16. Sol:

$$\bar{\alpha} \parallel (\bar{\beta} \times \bar{r}) \Rightarrow \bar{\alpha} \perp \bar{\beta} \text{ and } \bar{\alpha} \perp \bar{r}$$

$$\begin{aligned} & \text{as our } (\bar{\alpha} \times \bar{\beta}) \cdot (\bar{\alpha} \times \bar{r}) \\ &= \bar{\alpha} \cdot (\bar{\beta} \times (\bar{\alpha} \times \bar{r})) \\ &= \bar{\alpha} \cdot [(\bar{\beta} \cdot \bar{r})\bar{\alpha} - (\bar{\beta} \cdot \bar{\alpha})\bar{r}] \\ &= (\bar{\alpha} \cdot \bar{\alpha})(\bar{\beta} \cdot \bar{r}) - (0) \\ &= |\bar{\alpha}|^2 (\bar{\beta} \cdot \bar{r}) \end{aligned}$$

17. Sol:

$$\begin{aligned} \bar{f}\left(\frac{5}{4}\right) &= \left[\frac{5}{4}\right]\bar{i} + \left(\frac{5}{4} - \left[\frac{5}{4}\right]\right)\bar{j} + \left[\frac{5}{4} + 1\right]\bar{k} \\ &= \bar{i} + \left(\frac{5}{4} - 1\right)\bar{j} + 2\bar{k} \end{aligned}$$

$$\bar{f}\left(\frac{5}{4}\right) = \bar{i} + \frac{1}{4}\bar{j} + 2\bar{k}$$

$$0 < t < 1 \Rightarrow \bar{r}(t) = 0\bar{i} + (t-0)\bar{j} + \bar{k}$$

$$\bar{f}(t) = t\bar{j} + \bar{k}$$

$$\text{Let } \theta = \left(\bar{f}\left(\frac{5}{4}\right), \bar{f}(t) \right)$$

$$\begin{aligned} \cos \theta &= \frac{2 + \frac{t}{4}}{\sqrt{1 + \frac{1}{16} + 4\sqrt{1+t^2}}} \\ &= \left(\frac{8+t}{9\sqrt{1+t^2}} \right) \end{aligned}$$

18. Sol:

$$\begin{aligned} & \left(\bar{a} \times (\bar{a} \times (\bar{a} \times (\bar{a} \times \bar{b}))) \right) \\ &= \bar{a} \times (\bar{a} \times [(\bar{a} \cdot \bar{b})\bar{a} - (\bar{a} \cdot \bar{a})\bar{b}]) \\ &= -4(\bar{a} \times (\bar{a} \times \bar{b})) \\ &= -4[(\bar{a} \cdot \bar{b})\bar{a} - (\bar{a} \cdot \bar{a})\bar{b}] = 16\bar{b} \\ &= 16|\bar{b}| \left(\frac{\bar{b}}{|\bar{b}|} \right) = 16 \times 3\hat{b} = 48\hat{b} \end{aligned}$$

19. Sol:

$$4\bar{a} + 5\bar{b} + 9\bar{c} = \bar{0} \Rightarrow$$

$\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\Rightarrow \bar{b} \times \bar{c}, \bar{c} \times \bar{a}$ are collinear

$$\Rightarrow (\bar{b} \times \bar{c}) \times (\bar{c} \times \bar{a}) = \bar{0}$$

20. Sol:

$$[\bar{a} \quad \bar{b} \quad \bar{c}]^2 x^2 + 2[\bar{a} \quad \bar{b} \quad \bar{c}] + 1 = 0$$

$$\Rightarrow \Delta = 4[\bar{a} \quad \bar{b} \quad \bar{c}]^2$$

$$-4[\bar{a} \quad \bar{b} \quad \bar{c}]^2 = 0$$

roots are equal.

MATHS-B

1. $f(x)$ is defined for all $x \neq 0$. Now $f'(x) = \frac{a}{x} + 2bx + 1$

Since $x = -1$ and $x = 2$ are critical points

$$\text{we have } f'(-1) = 0 \Rightarrow -a - 2b + 1 = 0 \text{ and } f'(2) = 0 \Rightarrow \frac{a}{2} + 4b + 1 = 0$$

$$\text{Solving we get } a = 2, b = \frac{-1}{2}$$

2. Differentiating the given function we get $p'(x) = 2a_1x + 4a_2x^3 + \dots + (2n)a_nx^{2n-1}$ since all the powers of x are odd and coefficients are positive, $p'(x) \neq 0$ for all real $x \neq 0$ and $p'(0) = 0$ thus $x = 0$ is the only critical point and $p''(x)/x = 0 = 2a, > 0$

Hence $p(x)$ has only one minimum at $x = 0$

3. Let

$$\begin{aligned} f'(x) &= 25 \cdot x^{24} (1-x)^{75} - 75 \cdot x^{25} (1-x)^{74} \\ &= x^{24} (1-x)^{74} [25(1-x) - 75x] \\ &= x^{24} (1-x)^{74} (25 - 100x) \text{ also } 0 < x < 1 \text{ and} \end{aligned}$$

$$f'(x) = 0 \Rightarrow x = \frac{1}{4}$$

$$\text{Now } x < \frac{1}{4} \Rightarrow f'(x) > 0$$

$$x > \frac{1}{4} \Rightarrow f'(x) < 0$$

That is $f'(x)$ changes sign from positive to negative

$$\therefore f(x) \text{ has maximum at } x = \frac{1}{4}$$

4. Given function can be written as $f(x) = (x^4 - 1)^2 + (x^3 - 1)^2 + (x^2 - 1) + (x - 1)^2 + 5 \geq 5$

\therefore the minimum value of f is 5 and $\forall x \in R$ it attains its minimum value at $x = 1$

5. The given function can be written as

$$Z = (x + y + 1)^2 + (x - 2)^2 - 3 \geq -3 \quad x, y \text{ and equality hold when } x = 2, y = -3$$

\therefore minimum value of Z is -3

6. Let α, β be the roots

$$\therefore \alpha + \beta = a - 2; \alpha\beta = -(a+1)$$

$$\text{let } z = \alpha^2 + \beta^2$$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (a-2)^2 + 2(a+1)$$

$$= a^2 - 2a + 6$$

$$\frac{dz}{da} = 0 \Rightarrow 2a - 2 = 0$$

$$\Rightarrow a = 1 \quad \text{and} \quad \frac{d^2z}{da^2} = 2$$

Since, z is minimum at $x = 1$

7. Given function is continuous $\forall x \in R$

$$f'(x) = \frac{-2}{3}(x-1)^{-\frac{1}{3}}$$

So that $f'(1)$ does not exist also $f'(x) \neq 0$

For any value of $x \neq 1$ hence $x = 1$ is the only critical point for $f(x)$. Also

$$x < 1 \Rightarrow f'(x) > 0$$

$$x > 1 \Rightarrow f'(x) < 0$$

Thus $f'(x)$ changes sign from positive to negative at $x = 1$

$\therefore f$ is maximum at $x = 1$

8. Sol:

$$P(-1) = 10 \Rightarrow -a + b - c + d = 10 \rightarrow 1$$

$$P(1) = -6 \Rightarrow a + b + c + d = -6 \rightarrow 2$$

$$P'(x) = 3ax^2 + 2bx + c$$

$P'(x)$ has local minimum at $n=1$

$$\Rightarrow P'(1) = 0 \Rightarrow 6a + 2b = 0$$

$$\Rightarrow 3a + b = 0 \rightarrow 3$$

and $P(x)$ has maximum at $n = -1$

$$\Rightarrow P'(-1) = 0 \Rightarrow 3a - 2b + c = 0 \rightarrow 4$$

solving we get $a = 1, b = -3, c = -9, d = 5$

$$\therefore P(x) = x^3 - 3x^2 - 9x + 5$$

$$P'(x) = 3(x-3)(x+1)$$

$$P''(x) = 6x - 6$$

$$\text{if } P'(x) = 0$$

$$\Rightarrow x = 3, -1$$

$$P''(x) / x = 3 = 12 > 0$$

$$P''(x) / x = -1 = -12 < 0$$

$\therefore P(x)$ has maximum at $x = -1$ and minimum at $x = 3$.

Let A and B be points on the curve $y = p(x)$.

$$\therefore A = (-1, 10) ; B = (3, -22)$$

$$AB = \sqrt{1040} = \sqrt{16 \times 65} = 4\sqrt{65}.$$

9. Sol:

$$f(x) = 3x^2 - x + \frac{25}{12}$$

$$f'(x) = 6x - 1$$

$$f''(x) = 6$$

$$\text{If } f'(x) = 0 \Rightarrow x = \frac{1}{6}.$$

$\therefore f$ is minimum at $x = \frac{1}{6}$ and minimum value is $f\left(\frac{1}{6}\right) = 2$

let r be the common ratio. Then progression is r^2, r^3, r^4, \dots .

$$\Rightarrow \frac{r^2}{1-r} = 2$$

$$\Rightarrow r^2 + 2r - 2 = 0$$

$$r = \frac{2 \pm \sqrt{4+8}}{2} = -1 \pm \sqrt{3}$$

$$\therefore r = -1 + \sqrt{3} (\because |r| < 1)$$

10. Sol:

$$f(x) = \frac{1+x^2}{x+1} \quad (x \geq 0)$$

$$f'(x) = \frac{x^2 + 2x - 1}{(x+1)^2} = 1 - \frac{2}{(1+x)^2}$$

$$\text{If } f'(x) = 0 \Rightarrow (x+1)^2 - 2 = 0$$

$$x = \pm\sqrt{2} - 1$$

Now $x \geq 0 \Rightarrow x = \sqrt{2} - 1$ is only one critical point.

$$\text{Also } f''(x) = \frac{4}{(1+x)^3}$$

$$f''(\sqrt{2} - 1) = \frac{4}{2\sqrt{2}} > 0$$

$f(x)$ has least at $n = \sqrt{2} - 1$

$$\begin{aligned} \therefore f(\sqrt{2} - 1) &= \frac{1 + (\sqrt{2} - 1)^2}{1 + \sqrt{2} - 1} = 2\sqrt{2} - 2 \\ &= 2(\sqrt{2} - 1) \end{aligned}$$

11. Sol:

$$f(x) = \cos x \cdot \sqrt{\sin x}$$

$$f(0) = 0; f\left(\frac{\pi}{2}\right) = 0 \text{ and } f(x) > 0 \text{ for } 0 < x < \frac{\pi}{2}$$

$$f'(x) = -\sin x \sqrt{\sin x} + \cos x \cdot \left(\frac{1}{2\sqrt{\sin x}}\right) \cos x$$

$$= -(\sin x)^{3/2} + \frac{\cos^2 x}{2\sqrt{\sin x}}$$

$$= -(\sin x)^{3/2} \frac{\cos^2 x}{2\sqrt{\sin x}}$$

$$= \frac{-2\sin^2 x + \cos^2 x}{2\sqrt{\sin x}} = \frac{1 - 3\sin^2 x}{2\sqrt{\sin x}}$$

$$\text{if } f'(x) = 0 \Rightarrow \sin x = \frac{1}{\sqrt{3}}$$

$$\text{also } f'(x) > 0 \text{ when } \sin x < \frac{1}{\sqrt{3}}$$

$$\text{and } f'(x) < 0 \text{ when } \sin x > \frac{1}{\sqrt{3}}$$

$$\therefore f \text{ is maximum when } \sin x = \frac{1}{\sqrt{3}}$$

and maximum value is

$$f\left(\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) = \sqrt{1 - \frac{1}{3}} \cdot \sqrt{\frac{1}{\sqrt{3}}} = 2^{1/2} 3^{-3/4}$$

12. Sol:

$$f(x) = x^3 - 3x + 3$$

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$$

$$\text{if } f'(x) = 0 \Rightarrow x = \pm 1$$

$$f''(x) / x = 1 = 6 > 0$$

$$f''(x) / x = -1 = -6 < 0$$

$$\therefore s = \left\{ f(-3), f(-1), f(1), f\left(\frac{3}{2}\right) \right\}$$

$$= \left\{ -15, 5, 1, \frac{15}{8} \right\}$$

$$\therefore m = 5 \text{ and } m = -15$$

13. Sol:

$$f'(x) = 1 + \sec^2 x > 0$$

so if it is strictly increasing

$\therefore f$ has neither maximum (nor) minimum.

14. Sol:

$$f'(x) = \frac{1}{3} \sin \theta (3 \tan^2 x \sec^2 x) + (\sin \theta - 1) \sec^2 x$$

$$= \sin \theta \tan^2 x \cdot \sec^2 x + (\sin \theta - 1) \sec^2 x$$

$$f'(x) = 0 \Rightarrow \sin \theta \tan^2 x + \sin \theta - 1 = 0$$

$$\tan^2 x = \frac{1 - \sin \theta}{\sin \theta} < 0 \quad (\because \pi < \theta < 2\pi)$$

$\therefore f(x)$ has no critical points.

15. Sol:

let h be the height and r be the radius.

$$\therefore \pi r^2 h = K \text{ (constant)}$$

s be the total surface area of cylinder.

$$s = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r \left(\frac{k}{\pi r^2} \right) = 2\pi r^2 + \frac{2k}{r}$$

$$\frac{ds}{dr} = 4\pi r - \frac{2k}{r^2}$$

$$\text{Now } \frac{ds}{dr} = 0 \Rightarrow r^3 = \frac{k}{2\pi}$$

$$r = \left(\frac{k}{2\pi} \right)^{\frac{1}{3}}$$

$$\frac{d^2s}{dr^2} = 4\pi + \frac{4k}{r^3} > 0$$

$$\therefore s \text{ is minimum when } r = \left(\frac{k}{2\pi} \right)^{\frac{1}{3}}$$

$$\frac{h}{r} = \frac{\pi r^2}{r} = \frac{k}{\pi} \cdot \frac{1}{r^3} = \frac{k}{\pi} \cdot \frac{2\pi}{k} = 2$$

16. Sol:

dimensions of box are $3 - 2x, 8 - 2x, x$

$$V = x(3 - 2x)(8 - 2x)$$

$$\frac{dV}{dx} = 4(3x - 2)(x - 3)$$

$$\text{If } \frac{dV}{dx} = 0 \Rightarrow x = \frac{2}{3} \text{ and } x = 3. (\because x = 3 \text{ is impossible})$$

$$\left. \frac{d^2V}{dx^2} \right|_{x = \frac{2}{3}} = -28 < 0$$

$\therefore V$ has maximum at $x = \frac{2}{3}$

$$\text{maximum volume is } \frac{2}{3} \left(3 - \frac{4}{3} \right) \left(8 - \frac{4}{3} \right) = \frac{200}{27}$$

17. Sol:

$$s = x^2 + y^2 + z^2$$

$$= x^2 + 9x^2 + (26 - 4x)^2 (\because (y = 3x, z = 26 - x - y))$$

Now $z > 0$

$$\Rightarrow 26 - 4x > 0 \Rightarrow x < \frac{13}{2}$$

$\therefore s$ is a function of x on the interval $\left(0, \frac{13}{2} \right)$

$$\text{Now } \frac{ds}{dx} = 2x + 18x - 8(26 - 4x) = 0$$

$$\therefore 52x = 8 \times 26$$

$$x = \frac{8 \times 26}{52} = 4$$

$$\left. \frac{d^2s}{dx^2} \right|_{x = 4} > 0$$

$\therefore s$ is least at $x = 4$.

18. Sol:

$$h^2 + r^2 = l^2$$

$$v = \frac{1}{3}\pi r^2 h$$

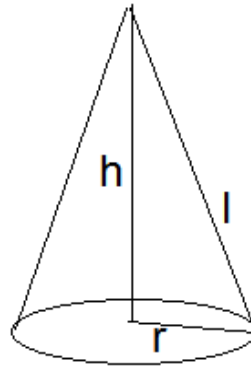
$$= \frac{1}{3}\pi h(l^2 - h^2)$$

$$\frac{dv}{dh} = \frac{\pi}{3}(l^2 - 3h^2)$$

$$\frac{dv}{dh} = 0 \Rightarrow h = \frac{l}{\sqrt{3}}$$

$$\frac{d^2v}{dh^2} = -2\pi h < 0 \text{ if } h = \frac{l}{\sqrt{3}}$$

$$\therefore v \text{ is maximum when } h = \frac{l}{\sqrt{3}}$$



19. Sol:

$$D = \text{distance } P \text{ to } Q = \sqrt{x^2 + (y+2)^2}$$

$$f(x) = D^2 = x^2 + \left(\frac{16}{\sqrt{3}x^3}\right)^2 = x^2 + \frac{256}{3x^6}$$

$$f'(x) = 2x - \frac{256}{3} \cdot (-6 \cdot x^{-7})$$

$$\text{if } f'(x) = 0 \Rightarrow x^8 = 256$$

$$\therefore x = 2 \quad f''(x)/x = 2 > 0$$

$f(x)$ has minimum at $x = 2$

$$\text{min value of } D \text{ is } \sqrt{4 + \left(\frac{16}{\sqrt{3}} \times \frac{1}{8}\right)^2} = \sqrt{4 + \frac{4}{3}} = \frac{4}{\sqrt{3}}$$

20. Sol:

Let r be the radius of base circle

h be the height of the cylinder

$$\therefore 2\pi = 2\pi r^2 + 2\pi rh$$

$$\Rightarrow r^2 + rh = 1$$

$$v = \pi r^2 h$$

$$= \pi r^2 \frac{(1-r^2)}{r}$$

$$= \pi(r - r^3)$$

$$\frac{dv}{dr} = 0 \Rightarrow \pi(1 - 3r^2) = 0$$

$$\Rightarrow r = \frac{1}{\sqrt{3}}$$

$$\frac{d^2v}{dr^2} \Big|_{r=\frac{1}{\sqrt{3}}} = -6\pi r < 0$$

$$\therefore v \text{ is maximum if } r = \frac{1}{\sqrt{3}}$$

$$\text{greatest volume is } \pi \left(\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}} \right)^3 \right)$$

$$= \pi \left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}} \right) = \frac{\pi}{\sqrt{3}} \left(\frac{2}{3} \right) = \frac{2\pi}{3\sqrt{3}}$$

PHYSICS

1. **Sol:**

$$\begin{aligned} \text{R.M.S Speed} &= \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \dots + V_n^2}{n}} \\ &= \sqrt{\frac{1^2 + 2^2 + 3^2 + 4^2}{4}} \\ &= \sqrt{\frac{30}{4}} \\ \bar{V}_{ms} &= \sqrt{\frac{15}{2}} \text{ km/s} \end{aligned}$$

2. **Sol:**

$$\text{Degrees of freedom of He } (f_{He}) = 3$$

$$\text{Degrees of freedom of O}_2 (f_{O_2}) = 5$$

$$\text{Degrees of freedom of O}_3 (f_{O_3}) = 6$$

$$n_{He} = 2, n_{O_2} = 4, n_{O_3} = 1$$

Energy of mixture = sum of individual energies

$$= (n_{He} f_{He} + n_{O_2} f_{O_2} + n_{O_3} f_{O_3}) \frac{RT}{2}$$

$$= (2 \cdot 3 + 4 \cdot 5 + 1 \cdot 6) \frac{RT}{2}$$

$$= (3 + 10 + 3) RT$$

$$= 16RT$$

3. **Sol:**

$$\gamma \text{ for mono atomic gas} = 1 + \frac{2}{3} = \frac{5}{3} = \gamma_1$$

$$\gamma \text{ for diatomic gas} = 1 + \frac{2}{5} = \frac{7}{5} = \gamma_2$$

$$\frac{n}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\frac{2}{\gamma - 1} = \frac{1}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1}$$

$$\text{Solving we get } \gamma = \frac{3}{2}$$

4. **Sol:**

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \quad T_1 = T, T_2 = T + 6$$

$$\frac{P_2}{P_1} = 1 = \frac{T_2}{T_1} - 1$$

$$\frac{P_2 - P_1}{P_1} \times 100 = \left(\frac{T + 6}{T} - 1 \right) 100$$

$$0.4 = \frac{600}{T} \Rightarrow T = 1500K$$

5. **Sol:**

$$PV = nRT$$

$$V \propto T$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \text{given } V_1 = V, T_1 = 27^\circ C = 300K$$

$$V_2 = 2V, T_2 = ?$$

$$\frac{V}{2V} = \frac{300}{T_2}$$

$$T_2 = 600K$$

$$T_2 = 600 - 273 = 327^\circ C$$

6. **Sol:**

$$\bar{V}_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

$$V_{avg} \propto \sqrt{T}$$

7. **Sol:**

$$\text{Number of moles of } N_2 = \frac{10}{28}$$

$$U = \frac{f}{2} nRT$$

$$= \frac{5}{2} \times \frac{5}{14} \times R \times 273$$

$$U = 2025T$$

8. **Sol:**

$$8g \text{ of } O_2 = \frac{1}{4} \text{ moles}$$

$$PV = nRT$$

$$PV = \frac{1}{4} RT$$

9. **Sol:**

$$\bar{V}_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V \propto \sqrt{T}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} \quad \text{given } V_1 = V, V_2 = ?$$

$$T_1 = 27 + 273 = 300K$$

$$T_2 = 927 + 273 = 1200K$$

$$\frac{V}{V_2} = \sqrt{\frac{300}{1200}} = \frac{1}{2}$$

$$V_2 = 2V$$

10. **Sol:**

Total number of molecules remains constant.

$$n_1 + n_2 = n_1^1 + n_2^1$$

$$\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T} + \frac{P_2 V_2}{T}$$

Solving we get

$$T = \frac{P_1 V_1 + P_2 V_2}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}$$

11. **Sol:**

$$R = \frac{L}{KA}$$

$$[R] = [M^{-1} L^2 T^3 K]$$

12. **Sol:**

$$H = \frac{K \pi r^2}{L} (T_2 - T_1)$$

$$H \propto r^2 \text{ and } H \propto \frac{1}{L}$$

So the rod with maximum $\frac{r^2}{L}$ will conduct maximum heat

13. **Sol:**

Incoming heat = outgoing heat

$$(90^\circ - \theta) + (60^\circ - \theta) = \theta - 30^\circ + \theta - 0^\circ$$

$$180^\circ = 4\theta$$

$$\theta = 45^\circ$$

14. **Sol:**

$$\frac{T - 20}{100 - 20} = \frac{60 - 0}{100 - 0}$$

$$\frac{T - 20}{80} = \frac{60}{100}$$

$$T = 48 + 20 = 68^\circ C$$

15. **sol:** kirchhoff's law states that "A good absorber is a good emitter"

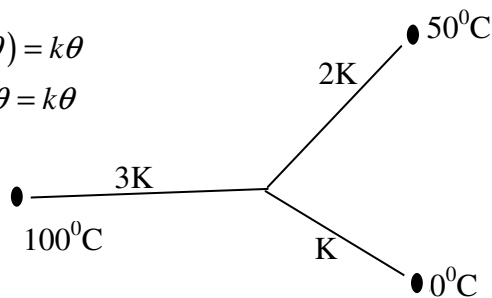
16. Sol:

$$3k(100 - \theta) + 2k(50 - \theta) = k\theta$$

$$300k - 3k\theta + 100k - 2k\theta = k\theta$$

$$400 = 6\theta$$

$$\theta = \frac{200}{3} ^\circ\text{C}$$



17. Sol:

Wien's law

$$\lambda_m T = \text{const } t$$

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$0.5 \times 10^{-6} \times T_1 = 10^{-4} \times T_2$$

$$\frac{T_1}{T_2} = 200$$

18. Sol:

$$t + r + a = 1$$

$$a = 1 - (t + r)$$

$$= 1 - \left(\frac{1}{9} + \frac{1}{6} \right)$$

$$a = \frac{13}{18}$$

19. Sol:

$$E \propto T^4 \quad \text{when } T = 727 + 273 = 1000\text{K}$$

$$E \propto (1000)^4$$

20. Sol:

$$80^\circ\text{C} \xrightarrow{t_1} 75^\circ\text{C} \xrightarrow{t_2} 70^\circ\text{C} \xrightarrow{t_3} 65^\circ\text{C}$$

According to Newton's law of cooling

Rate of cooling \propto mean temperature difference

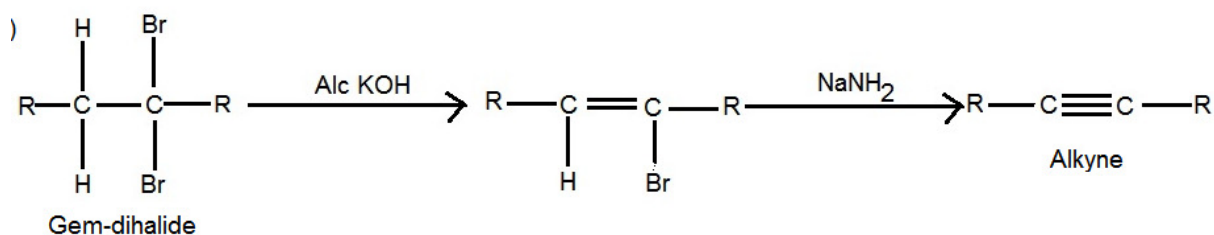
$$\frac{\text{fall in temperature}}{\text{Time}} \propto \left(\frac{\theta_1 + \theta_2}{\theta_2} - \theta_0 \right)$$

So cooling will be fastest in the first case and slowest in the last case (third case)

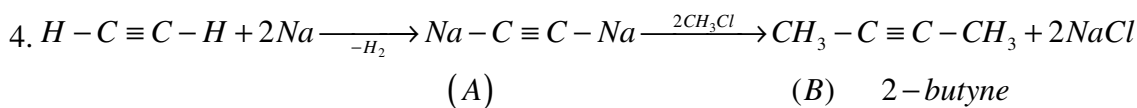
$$\therefore t_1 < t_2 < t_3$$

CHEMISTRY

1. Sol:

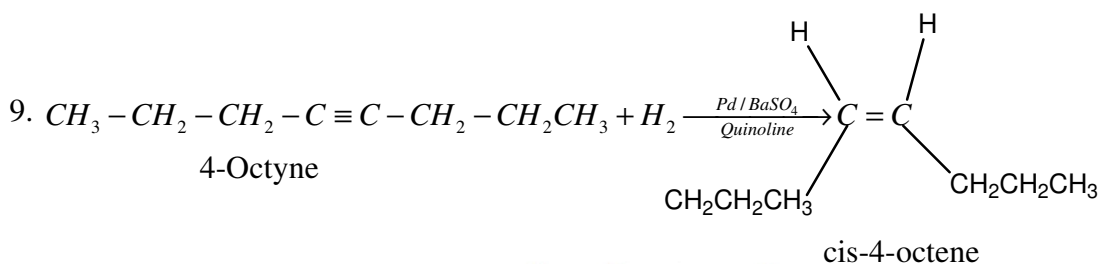
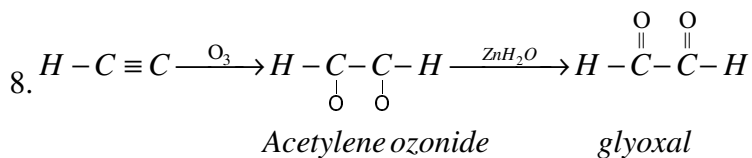
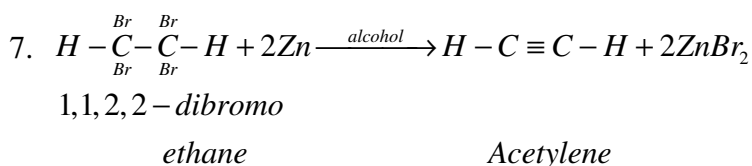
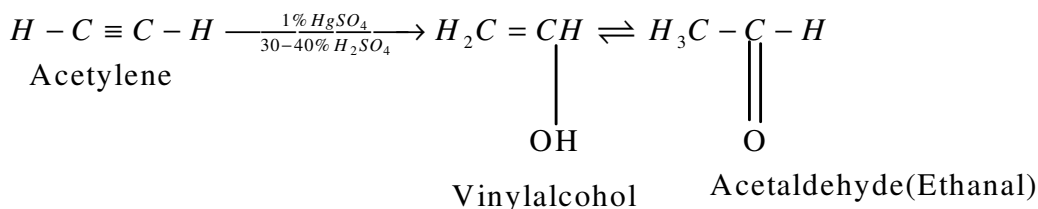


2. Acidic nature of terminal alkynes can be detected by silver mirror test.
 3. Na in liq. NH₃ converts substituted alkynes to trans alkenes (Birch reduction)



5. Alkynes are functional isomers of alkenes

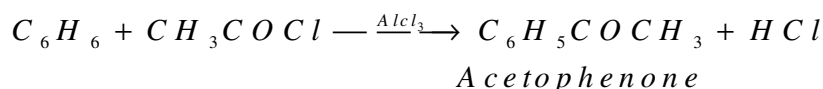
6.



10. C-C bond length is short in alkynes (H—C≡C—H)

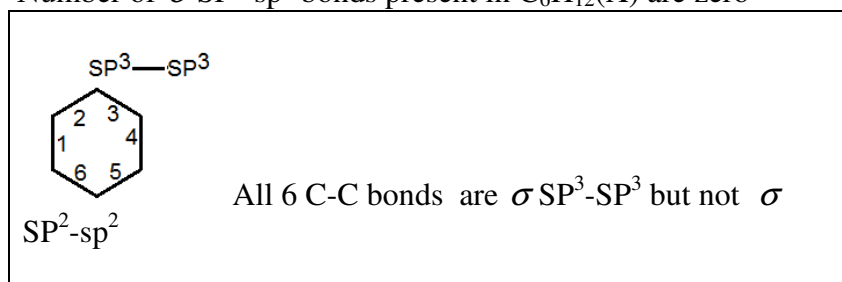
11. Preparation of benzene from phenol by using Zn is reduction

12.



13. Benzene reacts with Cl₂ in the presence of sunlight (hν) gives hexachlorocyclohexane (A) and in the presence of Lewis acid FeCl₃ gives chlorobenzene (C₆H₅Cl) (B)

14. Number of σ SP²-sp² bonds present in C₆H₁₂ (X) are zero



15. Cycloheptatrienyl cation is aromatic

16. -COOH group is metadirecting

17. due to resonance

18. 2-phenyl heptanes

19. Dipole moment of benzene is zero
20. HNO_3 acts as a base in nitration mixture