SRIGAYATRI EDUCATIONAL INSTITUTIONS

INDIA

Sec : SR MPC	GT-1	Date: 08-09-20
Time: 2:00 PM to 5:00 PM	2012_P2	Max.Marks:198

KEY SHEET

PHYSICS

1	**D**	2	**A**	3	**A**	4	**C**	5	**C**
6	**B**	7	**A**	8	**D**	9	**A**	10	**C**
11	**A**	12	**A**	13	**B**	14	**B**	15	**BD**
16	**BD**	17	**CD**	18	**AC**	19	**AB**	20	**CD**

CHEMISTRY

21	**B**	22	**C**	23	**A**	24	**C**	25	**B**
26	**A**	27	**C**	28	**C**	29	**A**	30	**C**
31	**D**	32	**B**	33	**A**	34	**B**	35	**D**
36	**AB**	37	**AB**	38	**BCD**	39	**AC**	40	**AD**

MATHS

41	**D**	42	**C**	43	**B**	44	**B**	45	**C**
46	**A**	47	**A**	48	**D**	49	**C**	50	**B**
51	**B**	52	**B**	53	**D**	54	**B**	55	**CD**
56	**A**	57	**ABCD**	58	**A**	59	**AB**	60	**BC**

SOLUTIONS

PHYSICS

1. Let m and d be the mass and diameter of the sphere, then the density ρ of the mass sphere is given by

$$\rho = \frac{mass}{volume} = \frac{m}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} = \frac{6m}{\pi d^3}$$

Taking log and differentiating partially we get

$$\frac{d\rho}{\rho} = \frac{dm}{m} - \frac{3d(d)}{d}$$

$\therefore$ maximum % error = 2+ (3×3)=11

2. To solve the problem you should have knowledge of conic section – parabola

The enveloping parabola is $x^2 = -4h(y - h)$where

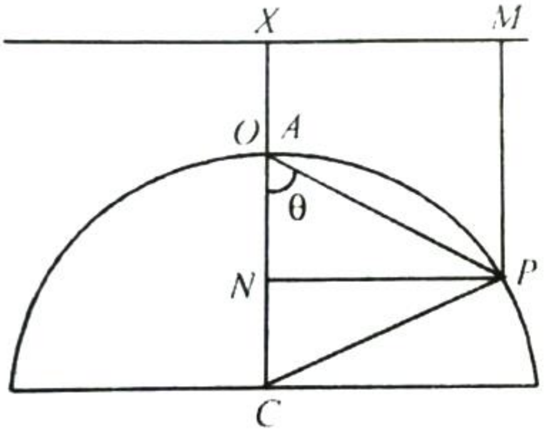
$$h = \frac{u^2}{2g}$$

Its focus is the point of projection O, vertex us A where OH=h and directix is XM

Let this enveloping parabola cut the sphere at P so that OP is the max. Range. Draw PM perp. To directrix and PN perpendicular to CO C the centre of the sphere. Since O is the focus, PO=PM=XN=XO+ON=2h+PO $\cos\theta$

Also $PO = 2r\cos\theta$. Hence, $PO\left(1 - \frac{PO}{2r}\right) = 2h$

Or $R\left(1 - \frac{R}{2r}\right) = 2h$ where R=PO



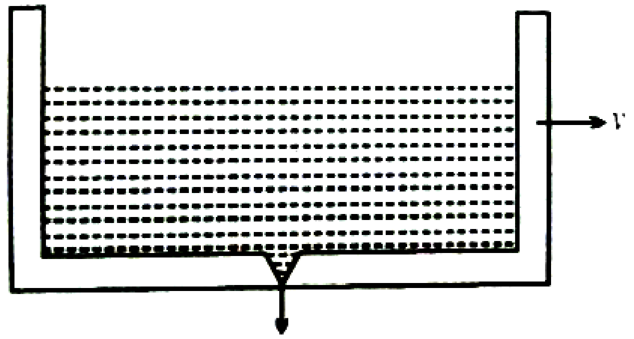
Or $R^2 - 2r + R + 4rh = 0$

$R = r \pm \sqrt{r^2 - 4rh}$

The plus sign corresponds to the second point where the parabola cuts the sphere again hence the minus sign is to be taken

$\therefore R = r - \sqrt{r^2 - 4rh}$

From this result is follows that the max. Value of h possible is r/4 when the two values of R become same and equal to r. In this case the parabola touches the sphere and the velocity of the projection is $\sqrt{2gh} = \frac{2g.\frac{r}{4}}{} = \sqrt{\frac{1}{2}gr}$. The particle will then clear the sphere and the least value of the velocity of projection must be $\sqrt{\frac{1}{2}gr}$

3. 

$$F_{thrust} + F_{ext} = m\frac{dv}{dt}; F_{ext} = 0$$

$$F_{thrust} = u_{rel} \times \frac{dm}{dt} \text{ and}$$

As $u_{rel} = 0 \Rightarrow F_{thrust} = 0$

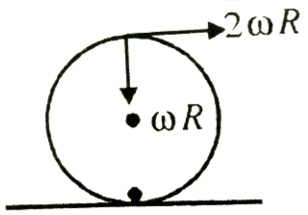
$$\Rightarrow \frac{mdv}{dt} = 0$$

$\Rightarrow$ constant velocity.

4. $p_f = p_i + \int_0^2 F.dt$

Total change in momentum = 10kg-,/s + $\int_0^2 (3 + 2t)dt$

$$p_f = 20kg - m/\sec; KE_f = \frac{p_f^2}{2m} = \frac{400}{4} = 100J$$

5. 

$$R = \frac{v^2}{a_\perp} = \frac{(2wR)^2}{w^2R} = 4R$$

6. $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5$

$$|\vec{F}_2| = |\vec{F}_5| \text{ and } |\vec{F}_2| = |\vec{F}_4|$$

$$\vec{F}_2 = \vec{F}_3 + 2F_2\cos 30^\circ + 2F_1\cos 60^\circ$$

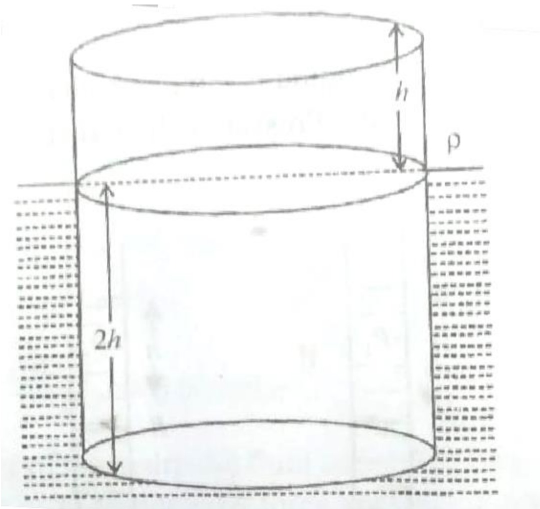
$$F_3 = \frac{Gm^2}{4a^2}; F_2 = \frac{Gm^2}{3a^2}; F_1 = \frac{Gm^2}{a^2}$$

$$F = \frac{Gm^2}{a^2}\left(\frac{5}{4} + \frac{1}{\sqrt{3}}\right) = mw^2 a$$

$$w = \sqrt{\frac{Gm}{a^2}\left(\frac{5}{4} + \frac{1}{\sqrt{3}}\right)}$$

$$T = 2p\sqrt{\frac{4\sqrt{3}a^3}{Gm\left(5\sqrt{3} + 4\right)}}$$

7. Let A be the area of the cross – section of the cylinder and 3h its height. Let r_1 be the density of water. Then the weight of the cylinder = $A.3hs r_1 g$



Then weight of air displaced = $Ah.r r_1 g$, since the length of cylinder in air is $\frac{1}{3}(3h)$i.e. h.

The weight of water displaced = $A.2hr_1 g$

Fir equilibrium, $A.3hs r_1 g = Ahr_1 g + A.2hr_1 g$ or $3s = r + 2$

8. $v_1 = w\sqrt{A^2 - y_1^2}$

And $v_2 = w\sqrt{A^2 - y_2^2}$

$v_2^2 = w^2 A^2 - w^2 y_2^2$ ____(1)

$v_1^2 = w^2 A^2 - w^2 y_1^2$ ____(2)

Solving 1 and 2, we get

$$v_2^2 - v_1^2 = w^2\left(y_1^2 - y_2^2\right)$$

$$w = \sqrt{\frac{v_2^2 - v_1^2}{y_1^2 - y_2^2}}$$

$$T = \frac{2p}{w} = 2p\sqrt{\frac{y_1^2 - y_2^2}{v_2^2 - v_1^2}}$$

9 & 10.

The equations are $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ represents two progressive wave travelling in the same direction with slight difference in the frequency. This will give the phenomenon of beats.

Comparing it with the equation

$y = A\cos(kx - \omega t)$, we get

$\omega_1 = 100\pi \Rightarrow 2\pi f_1 = 100\pi \Rightarrow f_1 = 50Hz$ and

$K_1 = 0.5\pi \Rightarrow \frac{2\pi}{\lambda_1} = 0.5\pi \Rightarrow \lambda_1 = 4m$

Wave velocity $= \lambda_1 f_1 = 200m/s$ [alternatively use $= \frac{\omega}{K}$]

$\omega_2 = 92\pi \Rightarrow 2\pi f_2 = 92\pi \Rightarrow f_2 = 46Hz$

Therefore beat frequency $= f_1 - f_2 = 4Hz$ and

$K_2 = 0.46\pi \Rightarrow \frac{2\pi}{\lambda_2} = 0.46\pi \Rightarrow \lambda_2 = \frac{200}{46}$

Wave velocity $= \frac{200}{46} \times 46 = 200m/s$

11. Process AB, U_ρ = constant

$\frac{P}{\rho} = \frac{RT}{M}$ and $U \propto t$

$\Rightarrow$ P= const

Process BC $\rightarrow$ isochoric

Process CA $\rightarrow$ isothermal

12. $Q = Q_{AB} + Q_{BC} + Q_{CA}$

$Q = -5U_0 + 3U_0 + \frac{10U_0}{3}\ln 2.5$

13 & 14.

$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \frac{Q}{2R} + \frac{3Q}{3R}\right)$

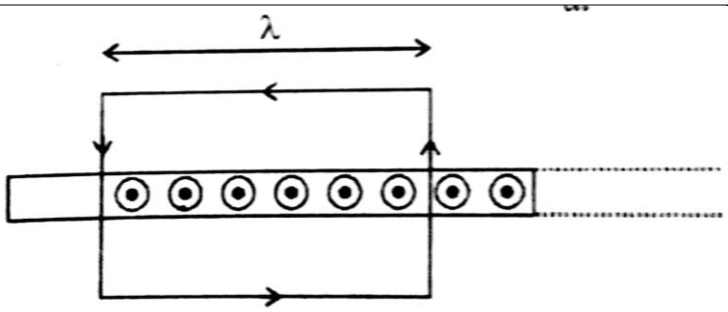
$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2R} - \frac{Q}{2R} + \frac{3Q}{3R}\right)$

$V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{3R} - \frac{Q}{3R} + \frac{3Q}{3R}\right)$

$\therefore 2V_A = 3V_B$

$V_{BC} = V_B - V_A = 0$

15. Current flowing per unit length $= \sigma \frac{dx}{dt} = \sigma v$



By Ampere's law

$Bl + Bl = \mu_0 \sigma v l \Rightarrow B = \frac{\mu_0 \sigma v}{2}$

16. Magnitude of induced electric field due to change in magnetic flux is given by

$\oint \vec{E}.d\vec{l} = \frac{d\phi}{dt} = A\frac{dB}{dt} \left(\because N = 1 \text{ and } \cos\theta = 1\right)$

Or $E.l = \pi R^2 (2B_0 t)\left(\frac{dB}{dt} = 2B_0 t\right)$

Here, E= induced electric field due to change in magnetic flux

$E(2\pi R) = 2\pi R^2 B_0 t$

Or $E = B_0 R t$

Hence, $F = \overline{QE} = B_0 Q R t$

This force is tangential to ring. Ring starts rotating when torque of this force is greater than the torque due to maximum friction $(f_{max} = \mu mg)$

Or when $\tau_F \geq \tau_{f_{max}}$

Taking the limiting case, $\tau_F = \tau_{f_{max}}$ or

$F.R = (\mu mg)R$

It is given that ring starts rotating after 2 seconds.

So, putting t=2 seconds, we get $\mu = \frac{2B_0 RQ}{mg}$

17. From first lens,

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-30} = \frac{1}{+15}$

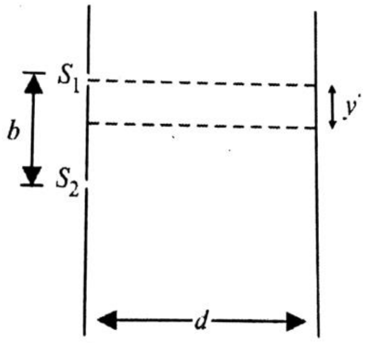
$\Rightarrow \frac{1}{v} = \frac{\frac{1}{15} - \frac{1}{30}}{30} = \frac{1}{30}$

$\Rightarrow v = +30cm$

Thus the first lens will form image at optical centre of lens L_2 and lens L_2 will form image at its optical itself because object distance is close to zero.

18. Here $y=(2n-1)\frac{\lambda}{2}\frac{D}{d}=(2n-1)\frac{\lambda}{2}\frac{d}{b}$

$(\because d=b\, and\, D=d)$



But $y=\frac{b}{2}$

$\therefore \frac{b}{2}=(2n-1)\frac{\lambda}{2}\frac{d}{b}$

19. The mass number decreases by 32 hence

$8\alpha-$particles and $6\beta-$particles are produced.

${}_{92}U^{238}\rightarrow {}_{82}Pb^{206}+8\,{}_{2}He^{4}+6\,{}_{-1}\beta^{0}$

20. $\frac{1}{\lambda_{K\alpha}}=RZ^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=\frac{3}{2}RZ^2$

$\frac{1}{\lambda_0}=\frac{eV_A}{hc}$

CHEMISTRY

21. I) Entropy of a system decreases with decreases in temperature and the system is expected to have the minimum value of entropy at zero Kelvin.

II) Only data about Δ_{Sys} is not sufficient.

III) $\frac{dT}{dP}=\frac{T\left(V_L-V_S\right)}{\Delta_f H}$

For substances like sulphur which expand on melting, $\left(V_L-V_S\right)$ will be positive and dT/dP is positive.

IV) $C_p-C=R\left(1+\frac{2a}{VRT}\right)$ for a gas obeying Vander Wall's equation and remembering that 'a' is small.

22. One and the same enzyme can catalyse the break down as well as the synthesis of the same substance.

23. I) Hyophobic > hyophilic (cataphoresis)

II) Brownian motion doesnot allow the colloidal particles to settle down due to gravity.

III) The concentration (or) pH at which the colloidal particles have no charge is known as iso electric point.

IV) In o/w emulsions, water is the continuous medium and aqueous systems are, in main good conductors.

24. D.C can't be used as this would result in electrolysis & concentration changes at electrodes. The gas liberated increases the internal resistance of electrolyte between electrodes. The gases also set up a counter EMF of "polarization" which opposes the passage of current.

25. A) $Z = 1 + \frac{bP}{RT}$ B) $P \geq P_C$

C) Yes, since otherwise, with time all the molecules will be collected in one direction.

D) $\lambda = \frac{1}{\sqrt{2} \times \pi\sigma^2 N^*}$.

26. A) Δt is always much smaller than Δ_0,

This it is never energetically favourable to pair electrons.

B) Tetrahedral complexes are favoured where the ligands are large and bulky.

C) Jahn – Teller theorem.

D) $Ba^{2+} + SO_4^{2-} \rightarrow BaSO_4$

27. Conceptual.
28. Conceptual.
29. Calcium.
30. With Fe^{3+} it forms a reddish – brown PPt which is insoluble in excess of the reagent (D)
31. D is $NaBO_2$
32. F is diborane and it undergoes unsymmetrical cleavage with primary amines.
33. Conceptual.
34. Conceptual.
35. A) NH_3 B) NH_3 C) N_2O D) N_2
36. A) $Mg(NH_4)PO_4$ B) BF_3

C) No nitrate is formed D) $(NH_4)_2 PtCl_6$ *yellow PPt*

37. A) $HNO_3 + 2HF \rightarrow H_2NO_3^+ + HF_2^-, H_2SO_4 + 2HF \rightarrow H_3SO_4^+ + HF_2^-$.

B) $NH_3 \rightleftharpoons NH_2^- + H^+$.

C) $N^{3-} + 2NH_3 \rightarrow 3NH_2^-$ actidity decreases.

D) Greater back bonding in BF_2Cl.

38. A) C – O bond lengths in CO is 1.128A°.

C – O bond lengths in CO^+ is 1.115A°.

B) $1\sigma + \frac{1\pi}{3}$

C) $N_3^- : 16_e^-$

D) Apply M.O.T

39. Conceptual.

40. Conceptual.

MATHS

41. $\tan 12 + \tan 48 + \tan 84 + \tan 120 + \tan 156 \quad = \frac{\sin 60}{\cos 12 \cos 48} - \frac{\sin 60}{\cos 24 \cos 84} + \tan 120$

$= \frac{2\sin 60}{\cos 60 + \cos 36} - \frac{2\sin 60}{\cos 108 + \cos 60} - \sqrt{3} = 5\sqrt{3}$

42. $I = \int_0^\infty \frac{x^2 + ax + 1}{1 + x^4} \tan^{-1}\left(\frac{1}{x}\right) dx$

put $x = \frac{1}{t} dt$

$I = \int_0^\infty \frac{x^2 + ax + 1}{1 + x^4} \tan^{-1} x \, dx$

$2I = \frac{\pi}{2}\int_0^\infty \frac{x^2 + ax + 1}{1 + x^4} dx = \frac{\pi}{2}\int_0^\infty \frac{x^2 + 1}{1 + x^4} dx + \frac{\pi a}{2}\int_0^\infty \frac{x}{1 + x^4} dx$

43. $3 + (m-1)8 = 7 + (n-1)23$

$8m - 5 = 23n - 16$

$23n - 8m = 11$

$n = \frac{11 + 8m}{23} = 1 + \frac{4(2m-3)}{23}$

$m = 13, \ n = 5$

$m = 13, \ n = 13$

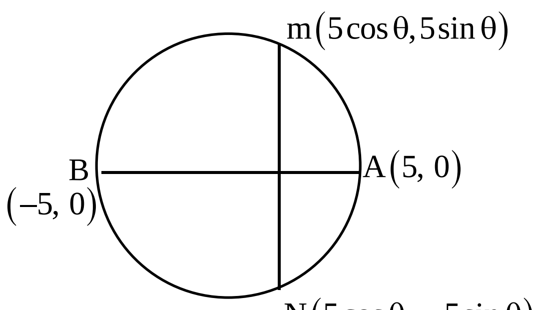
$n = 21$

$n = 29$

$n = 37$

when n =13, 21, 29, 37 we get 3 digit numbers

44.



Equation of BM is

$y = \frac{5\sin\theta(x+5)}{5(\cos\theta + 1)}$

$y=\tan\frac{\theta}{2}(x+5)$ 1

similarly equation of AN is

$y=\cot\frac{\theta}{2}(x-5)$ 2

1 x 2

$y^2=x^2-25$

$x^2-y^2=25$

$\Rightarrow e=\sqrt{2}$

45. $\underset{x\to\alpha}{Lt}\frac{f^{-1}(x)}{x^{\frac{1}{3}}}$ $x=f(t)$ As $x\to\alpha,\ t\to\alpha$

$=\underset{t\to\alpha}{Lt}\frac{t}{(f(t))^{\frac{1}{3}}}=\underset{t\to\alpha}{Lt}\frac{t}{(8t^3+3t)^{\frac{1}{3}}}=\frac{1}{2}$

similarly $\frac{f^{-1}(8x)}{x^{\frac{1}{3}}}, 8x=f(t)$

$=\underset{t\to\alpha}{Lt}\frac{t}{\left(\frac{f(t)}{8}\right)^{\frac{1}{3}}}=2.\frac{1}{2}=1$

$\therefore\ \frac{1}{1-\frac{1}{2}}=2$

46. Let n(A) be the number of ways first person does n't get any letter. similarly n(B), n(C), n(D),

Number of ways such that everyone gets alteast one letter =

$n(\overline{A}\cap\overline{B}\cap\overline{C}\cap\overline{D})=n(\mu)-n(A\cup B\cup C\cup D)$

$={}^7C_3\times4\times4\times4-4.\ {}^6C_2\times3\times3\times3$

$+{}^4C_2\times{}^5C_1\times2\times2\times2-{}^4C_3\times{}^4C_0\times1\times1\times1$

= 856

Number of ways we can distribute so that each gets atleast one letter and one person gets all 4A's = 24

47. All those lines lie on the plane containing the point (1, 1, 1) and dr's of the normal being 1,2,3

So the equation of the plane is $x+2y+3z=6$,

Intersection of planes $x+y+z=1$ and $x+2y+3z=6$ is $\frac{x+4}{1}=\frac{y-5}{-1}=\frac{2}{1}$

Now find the minimum distance between (1, 1, 1) and the line

48. when x = 0; $a+a^2\ge2$

$\Rightarrow a\le-2\ (becuase\ a<0)$

$a^2+a\cos x\ge a^2+a\ge2$ (when a = - 2)

$a^2+a\cos x\ge2\ge\cos^2x+\cos x$

$\sin^2x+a\cos x+a^2\ge1+\cos x$

Hence the range of values for negative a is a ≤ -2

49. $I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{x+5+\sqrt{x^2+25}} dx = \int_{-\pi}^{\pi} \frac{\sin^2 x}{-x+5+\sqrt{x^2+25}} dx$

$$2I = \int_{-\pi}^{\pi} \frac{\sin^2 x \quad 2\left(5+\sqrt{x^2+25}\right)}{\left(\sqrt{x^2+25}+5\right)^2 - x^2} dx$$

$$= \frac{2}{5}\int_{-\pi}^{\frac{\pi}{2}} \sin^2 x\, dx$$

$$I = \frac{2}{5}\int_{0}^{\frac{\pi}{2}} \sin^2 x\, dx$$

$$I = \frac{2}{5}.\frac{1}{2}.\frac{\pi}{2} = \frac{\pi}{10}$$

50. $I = \int_{0}^{\frac{\pi}{2}} \frac{\cos x}{\left(1+\sqrt{\sin 2x}\right)^2} dx$

$$I = \int_{0}^{\frac{\pi}{2}} \frac{\sin x}{\left(1+\sqrt{\sin 2x}\right)^2} dx$$

$$2I = \int_{0}^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\left(1+\sqrt{\sin 2x}\right)^2} dx$$

$\sin x - \cos x = t$

$$2I = \int_{-1}^{1} \frac{dt}{\left(1+\sqrt{1-t^2}\right)^2}$$

$t = \sin\theta$

$$I = \int_{0}^{1} \frac{\cos\theta d\theta}{4\cos^4 \frac{\theta}{2}}$$

$$= \int_{0}^{\frac{\pi}{2}} \frac{2\cos^2 \frac{\theta}{2} - 1}{4\cos^4 \frac{\theta}{2}} = \int_{0}^{\frac{\pi}{2}} \frac{1}{2}\sec^2\frac{\theta}{2} - \frac{1}{4}\int_{0}^{\frac{\pi}{2}} \sec^4\frac{\theta}{2}$$

51,52.

Let O_1, O_2 be the centres of given circles BC = 4 and $O_1\ O_2$ =4 and $O_1\ O_2$ BC is a rectangle also ABC is an equilateral triangle

$\frac{AP_1}{P_1B} = \frac{4-\sqrt{3}}{\sqrt{3}}$ and the ordinate of new position of A is $2-2\sqrt{3}$

53, 54.

$$z = \cos^2 + \sin^2 t + \sin^4 t + i\left(a\cos^4 t + \frac{b}{2}\sin^2 2t + c\sin^4 t\right)$$

$x = \sin^2 t$

$y = a\cos^4 t + 2b\cos^2 t + \sin^2 t + c\sin^4 t$

$= a(1-x)^2 + 2bx(1-x) + cx^2$

$y = (a+c-2b)x^2 + 2(b-a)x + a$

If $a+c=2b$ then A,B,C will be collinear

$0 \le x \le 1$

56. $z_1 = x(\cos A + i\sin A)$

$z_2 = y(\cos B + i\sin B)$

$z_3 = z(\cos C + i\sin C)$

$\text{Im}(z_1+z_2+z_3) = 0$

$\Rightarrow z_1+z_2+z_3$ is real

$z_1^2+z_2^2+z_3^2$ is real

similarly $z_1z_2+z_2z_3+z_3z_1$ is real

$z_1^3+z_2^3+z_3^3-3z_1z_2z_3$

$=(z_1+z_2+z_3)(z_1^2+z_2^2+z_3^2-z_1z_2-z_2z_3-z_3z_1)$

$z_1z_2z_3 = xyz\,e^{i(A+B+C)}$ is real

$\Rightarrow z_1^3+z_2^3+z_3^3$ is real

57. $A^3 - pA^2 + p^2A = 0$ 1

$A^2 - pA + p^2AI = 0$

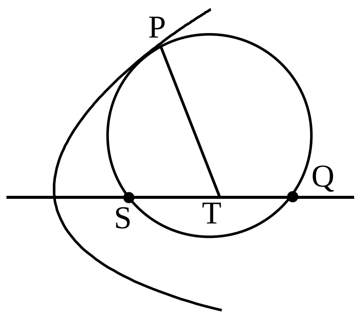
$pA^2 - p^2A + p^3I = 0$ 2

$A^3 = -p^3I$

$|A|^3 = -p^3$

Trace of AA^T is always positive irrespective of the elements in A.

60.



$P(at^2, 2at)$

PT is normal

T is $(2a+at^2, 0)$

Q is $(3a+2at^2, 0)$

$SP \perp PQ$