


SR MPC JEE MAINS (UT1) – QUESTION BANK
WAVES
Waves (Multiple type)

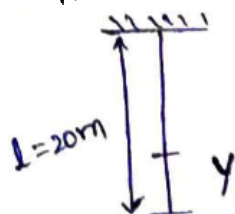
1. Uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is ($g = 10 \text{ m/s}^2$)

1) $2\pi\sqrt{2} \text{ S}$ 2) 2 S 3) $2\sqrt{2} \text{ S}$ 4) $\sqrt{2} \text{ S}$

Ans:3

Sol: Speed of the wave pulse in the string

$$v = \sqrt{\frac{T}{\mu}}$$



$$T = \frac{m}{l} \times y \times g$$

$$\mu = \frac{m}{l}$$

$$v = \sqrt{g y}$$

$$v = \frac{dy}{dt} = \sqrt{g y}$$

$$\int_0^{20} \frac{dy}{dt} = \int_0^t \sqrt{g} dt$$

$$t = 2\sqrt{2} \text{ S}$$

2. A tuning fork vibrates with frequency 256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe?

(Speed of sound in air is 340 m/s)

1) 190 cm 2) 180 cm 3) 200 cm 4) 220 cm

Ans:3

Sol: Organ pipe will have frequency either 255 Hz or 257 Hz

For frequency of tuning fork 255 Hz

$$255 = \frac{3v}{2l}$$

$$l = 2m$$

$$l = 200 \text{ cm}$$

3. When a stretched wire and a tuning fork are sounded together, 5 beats per second are produced, when length of wire is 95 cm or 100 cm, frequency of the fork is

1) 90 Hz 2) 100 Hz 3) 105 Hz 4) 195 Hz

Ans:4

Sol: Let the frequency of tuning fork be n .

Suppose, $l_1 = 95 \text{ cm}$ has frequency n_1

$l_2 = 100 \text{ cm}$ has frequency $n_2 (< n_1)$

$$n_1 - n = 5 \quad (1)$$

$$n - n_2 = 5 \quad (2)$$

Adding $n_1 - n_2 = 10 \quad (3)$

$$\frac{n_1}{n_2} = \frac{l_2}{l_1} = \frac{100}{95}$$

$$n_1 = \frac{100}{95} n_2$$

From (3) $n_2 = 190$

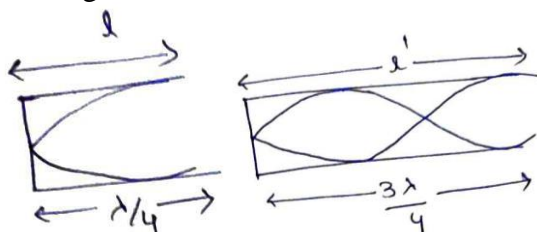
From (2) $n = 195 \text{ Hz}$

4. An air column, closed at one end open at the other, resonates with a tuning fork when the smallest length of one column is 50cm. The next larger length of the column resonating with the same tuning fork is

- 1) 150 cm 2) 200 cm 3) 66.7 cm 4) 100 cm

Ans:1

Sol: From fig



First harmonic is $l = \lambda / 4 = 50 \text{ cm}$

Third harmonic $l' = 3\lambda / 4 = 3 \times 50 = 150 \text{ cm}$

5. The two nearest harmonics of a tube closed at one end and open at other end are 220Hz and 260 Hz. What is the fundamental frequency of the system?

- 1) 20Hz 2) 30 Hz 3) 40 Hz 4) 10 Hz

Ans:1

Sol: Nearest harmonics of an organ pipe closed at one end differ by twice of its fundamental frequency

$$260 - 220 = 2g$$

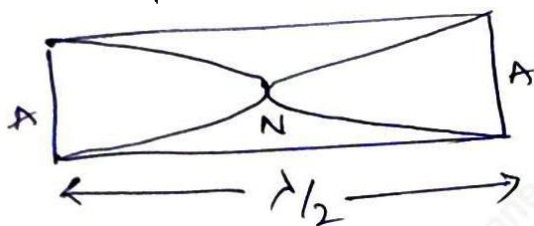
$$g = 20 \text{ Hz}$$

6. A granite rod of 60cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3 \text{ kg m}^{-3}$ and its young's modules is $9.27 \times 10^{10} \text{ gm}$ what will be the fundamental frequency of the longitudinal vibrations?

- 1)5KHz 2)2.5KHz 3)10KHz 4)7.5KHz

Ans:1

Sol: $v = \sqrt{\frac{y}{\rho}}$



$$= 5.86 \times 10^3 \text{ m/s}$$

Rod is clamped at the middle, shape of fundamental wave is as follows

$$\frac{\lambda}{2} = L$$

$$\lambda = 2L$$

$$L=60\text{cm}=0.6\text{m}$$

$$\lambda = 1.2\text{m}$$

$$v = \frac{V}{\lambda} = \frac{5.86 \times 10^3}{1.2} = 4.88 \times 10^3 \text{ Hz}$$

$$= 5\text{KHz}$$

7. If V_{rms} is the rms speed of molecules in a gas and V is the speed of sound waves in the gas, then the ratio $\frac{V_{rms}}{V}$ is

1) $\sqrt{\frac{3}{\gamma}}$ 2) $\sqrt{\frac{\gamma}{3}}$ 3) $\sqrt{3\gamma}$ 4) $\frac{\sqrt{3}}{\gamma}$

Ans:1

Sol: $V_{rms} = \sqrt{\frac{3p}{\rho}}$ -----(1)

Speed of sound in the gas is $v = \sqrt{\frac{\gamma p}{\rho}}$ -----(2)

$$\frac{(1)}{(2)} \Rightarrow \frac{V_{rms}}{v} = \sqrt{\frac{3}{\gamma}}$$

8. The ratio of the velocity of sound in hydrogen ($\gamma = \frac{7}{5}$) to that in helium ($\gamma = \frac{5}{3}$) at the same temperature is

1) $\sqrt{\frac{5}{42}}$ 2) $\sqrt{\frac{5}{21}}$ 3) $\frac{\sqrt{42}}{5}$ 4) $\frac{\sqrt{21}}{5}$

Ans:3

Sol: $V = \sqrt{\frac{\gamma RT}{M}}$

$$V \propto \sqrt{\frac{\gamma}{M}} \quad \frac{v_{Hz}}{v_{He}} = \sqrt{\frac{\gamma_{Hz}}{\gamma_{He}} \times \frac{M_{He}}{M_{Hz}}} = \sqrt{\frac{7}{5} \times \frac{3}{5} \times \frac{4}{2}} = \frac{\sqrt{42}}{5}$$

9. If the bulk modulus of water is 2100MPa, what is the speed of sound in water?

1) 1450m/s 2) 2100m/s 3) 1400m/s 4) 1200m/s

Ans:1

Sol: $v = \sqrt{\frac{B}{\rho}}$

$$v = \sqrt{\frac{2100 \times 10^6}{10^3}} = 1450 \text{ m/s}$$

10. String A has a length l , radius of cross-section r , density of material ρ and is under tension T , string B has all these quantities double those of string A. If v_A and v_B are the corresponding fundamental frequencies of the vibrating string, then

1) $v_A = 2v_B$ 2) $v_A = 4v_B$ 3) $v_B = 4v_A$ 4) $v_A = v_B$

Ans:2

Sol: $v = \frac{1}{2l} \sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{2Lr} \sqrt{\frac{T}{\rho}}$

$$\frac{v_A}{v_B} = \frac{L_B}{L_A} \times \frac{\gamma_B}{\gamma_A} \times \left(\frac{T_A \rho_B}{T_B \rho_A} \right)^{1/2}$$

$$v_A = 4v_B$$

11. The frequency of tuning fork is 256Hz. It will not resonate with a fork of frequency
 1) 768 Hz 2) 738 Hz 3) 512 Hz 4) 256 Hz

Ans: 2

Sol: Tuning fork of 256 Hz will resonate with fork of frequencies $1 \times 256, 2 \times 256, 3 \times 256$ etc.
 256Hz, 512Hz, 768Hz etc

Fork of 738 Hz will not resonate.

12. Two open organ pipes of fundamental frequencies ν_1 and ν_2 are joined in series. The fundamental frequency of the new pipe so obtained will be

1) $\nu_1 + \nu_2$ 2) $\frac{\nu_1 \nu_2}{\sqrt{\nu_1 + \nu_2}}$ 3) $\frac{\nu_1 \nu_2}{\nu_1 + \nu_2}$ 4) $\sqrt{\nu_1^2 + \nu_2^2}$

Ans: 3

Sol: $\nu_1 = \frac{v}{2L_1}$

$\nu_2 = \frac{v}{2L_2}$

Two pipes are joined in series new fundamental frequency

$$\nu = \frac{v}{2(L_1 + L_2)} = \frac{v}{2L_1 + 2L_2}$$

$$= \frac{v}{\frac{v}{\nu_1} + \frac{v}{\nu_2}} = \frac{\nu_1 \nu_2}{\nu_2 + \nu_1}$$

13. A sound wave travels with a velocity of 300m/s through a gas a beats are produced in 3s. when two waves pass through it simultaneously. If one of the waves has 2m wavelength, the wavelength of the other wave is

1) 1.98m 2) 2.04m 3) 2.06m 4) 1.99m

Ans: 2

Sol: No of beats Per sec = $\frac{9}{3} = 3s^{-1}$

$3 = \frac{\nu}{\lambda_1} - \frac{\nu}{\lambda_2}$

$3 = \nu \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$

$\frac{3}{300} = \frac{1}{2} - \frac{1}{\lambda_2}$

$\lambda_2 = 2.04m$

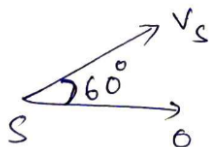
14. When two sound sources of the same amplitude but of slightly different frequencies ν_1 and ν_2 are sounded simultaneously, the sound one hears has a frequency equal to

1) $\nu_1 - \nu_2$ 2) $\frac{\nu_1 + \nu_2}{2}$ 3) $\sqrt{\nu_1 \nu_2}$ 4) $\nu_1 + \nu_2$

Ans: 2

Sol: The resulting sound wave has a frequency equal to half the sum of the individual frequencies. Note that the resulting intensity varies at the beat frequency equal to difference of the individual frequencies.

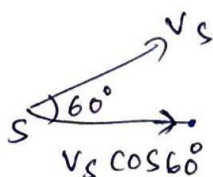
15. A source of sound S emitting waves of frequency 100Hz & an observer O are located at some distance from each other. The source is moving with a speed of 19.4m/s at angle 60° with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer (velocity of sound in air 330 m/s) is



- 1) 106Hz 2) 97Hz 3) 100Hz 4) 103Hz

Ans: 4

Sol:



$$v_0 = 100\text{Hz}$$

$$v_s = 19.4\text{m/s}$$

$$v = 330\text{m/s}$$

$$v = v_0 \left(\frac{v}{v - v_s \cos 60^\circ} \right)$$

$$100 \left(\frac{330}{330 - 19.4 \times \frac{1}{2}} \right)$$

$$= 103\text{Hz}$$

16. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660Hz. A listener is moving from source S_1 towards S_2 with a constant speed $u\text{m/s}$ and he hears 10 beats/s. The velocity of sound is 330m/s. Then u equals

- 1) 10.0 m/s 2) 2.5 m/s 3) 15.0 m/s 4) 5.5 m/s

Ans: 2

Sol: By Doppler's principle

$$\text{Beat frequency} = \frac{2vv_0}{v}$$

$$v_0 = \text{Velocity observer} = 4\text{ m/s}$$

$$v = 660\text{Hz, Beat frequency} = 10\text{Hz}$$

$$v = \text{Velocity of sound} = 330\text{m/s}$$

$$10 = \frac{2 \times 660 \times u}{330}$$

$$u = 2.5\text{m/s}$$

17. The phase difference between oscillatory motion of two points separated by a distance of $\frac{\lambda}{2}$ is (where λ is the wavelength)

- 1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{4}$ 4) 2π

Ans:2

Sol: when $\Delta x = \frac{\lambda}{2} \therefore \Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$

18. A transverse wave is represented by $y = A \sin(\omega t - kx)$. For what value of the wavelength is the wave velocity equal to the maximum particle velocity?

- 1) $\frac{\pi A}{2}$ 2) πA 3) $2\pi A$ 4) A

Ans:3

Sol: $y = A \sin(\omega t - kx)$

$$v = \frac{\omega}{k}$$

$$v_p = \frac{dy}{dt} = A\omega \cos(\omega t - kx)$$

$$(v_p)_{\max} = A\omega$$

$$\frac{\omega}{k} = A\omega$$

$$\left(k = \frac{2\pi}{\lambda} \right)$$

$$\frac{\lambda}{2\pi} = A$$

$$\lambda = 2\pi A$$

19. Two identical sinusoidal waves each of amplitude 10mm with a phase difference of 90° are travelling in the same direction in a string. The amplitude of the resultant wave is

- 1) 5 mm 2) $10\sqrt{2}$ 3) 15 mm 4) 20 mm

Ans:2

Sol: The two waves are identical, they have same amplitude with phase difference of 90°

$$A = \sqrt{a^2 + b^2} = \sqrt{a^2 + a^2} = a\sqrt{2}$$

$$a = 10\text{mm}$$

$$A = 10\sqrt{2}\text{mm}$$

20. In case of a travelling wave, the reflection at a rigid boundary will take place with a phase change of

- 1) $\frac{\pi}{2}$ radian 2) $\frac{\pi}{4}$ radian 3) $\frac{\pi}{6}$ radian 4) π radian

Ans:4

Sol: The reflection at a rigid boundary will take place with a phase reversal or with a phase change π or 180°

Waves Integer types

21. A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 90KHz. During one fast swoop directly toward a flat wall surface, the speed of moving at 0.06 times the speed of sound in air. what frequency does the bat hear reflected off the wall? _____

Ans: 90.21Hz

Sol: Ultrasonic beep frequency emitted by the bat

$$f = 80\text{KHz}$$

$$\text{Velocity of the bat } v_b = 0.06v$$

V=velocity of sound in air

The apparent frequency of the sound striking the wall is given by

$$\begin{aligned}
 v^1 &= \left(\frac{v}{v - v_b} \right) v \\
 &= \left(\frac{v}{v - 0.06v} \right) \times 80 \text{ KHz} \\
 v^1 &= \left(\frac{80}{1 - 0.06} \right) \text{ KHz}
 \end{aligned}$$

This frequency is reflected by the stationary wall ($v_1 = 0$) toward the bat.

The frequency (v'') of the received sound is given by the relation

$$\begin{aligned}
 v^{11} &= \left(\frac{v + v_b}{v} \right) v^1 \\
 &= \left(\frac{v + 0.06v}{v} \right) \times \frac{80}{1 - 0.06} \\
 v^{11} &= \frac{84.8}{0.94} = 90.21 \text{ KHz}
 \end{aligned}$$

22. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse(s) and longitudinal (p) sound waves. Typically the speed of S wave is about 14.0 km s^{-1} , and that of P wave is 28.0 km s^{-1} . A seismograph records P and S waves from an earthquake. The first P wave arrives 8 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Ans: 13,440 km

Sol: Here speed of S wave = $v_s = 14.0 \text{ km s}^{-1}$

Speed of P wave $v_p = 28.0 \text{ km s}^{-1}$

Time gap between P&S waves reaching the radiograph

$$T = 8 \text{ min}$$

Let distance of earthquake centre = $S \text{ km}$

$$\begin{aligned}
 t &= t_s - t_p \\
 &= \frac{S}{v_s} - \frac{S}{v_p} = \frac{S}{14.0} - \frac{S}{28.0} \\
 480 &= \frac{3}{28.0}
 \end{aligned}$$

$$S = 480 \times 28.0$$

$$= 13,440 \text{ km}$$

23. A SONAR system fixed in a submarine operates at a frequency 20 KHz. An enemy submarine moves towards the SONAR with a speed of 540 km h^{-1} . What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1200 ms^{-1} _____.

Ans: 25.71 KHz

Sol: Frequency of sonar (source) = 20 KHz

$$= 20 \times 10^3 \text{ Hz}$$

Speed of sound wave $v = 1200 \text{ ms}^{-1}$

Speed of observer $v_0 = 540 \text{ km h}^{-1}$

$$= 540 \times \frac{5}{18} = 150 \text{ ms}^{-1}$$

∴ The source is at rest & observer moves towards the source (SONAR)

$$v^1 = \frac{v + v_0}{v} \cdot g = \frac{1200 + 150}{1200} \times 20 \times 10^3$$

$$= \frac{1350}{1200} \times 20 \times 10^3$$

$$= 22.5 \times 10^3 \text{ Hz}$$

This frequency (ν^1) is reflected by the enemy ship & is observed by the SONAR (which now acts as observer).

In this case $= v_s = 540 \text{ km/h}$
 $= 150 \text{ m/s}$

As apparent frequency $\nu^{11} = \frac{v}{v - v_s} \nu^1$

$$= \frac{1200}{1200 - 150} \times 22.5 \times 10^3$$

$$= \frac{1200}{1050} \times 22.5 \times 10^3$$

$$= 25.71 \times 10^3 \text{ Hz}$$

$$= 25.71 \text{ KHz}$$

24. One end of a long string of linear mass density $10 \times 10^{-3} \text{ kg/m}$ is connected to an electricity driven tuning fork of frequency 512 Hz . The other end passes over a pulley and is tied to a pan containing a mass 100 kg . The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t=0$, the left end (fork end) of the string $x=0$ has zero transverse displacement along positive y -direction. The amplitude of wave is 10.0 cm write down the transverse displacement y as function of x and t that describes the wave on the string.....

Ans: $\nu(x, t) = 0.1 \sin(3215.3t - 10.27x)$

Sol: linear density $\mu = 10 \times 10^{-3} \text{ kg/m}$

$T = 100 \text{ kg} = 100 \times 9.8 \text{ N} = 980 \text{ N}$

$A = 10.0 \text{ cm} = 0.1 \text{ m}$

$\nu = 512 \text{ Hz}$

The wave propagating along the string is a transverse travelling wave, the velocity of the wave.

$$\nu = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{980}{10 \times 10^{-3}}} = \sqrt{98 \times 10^3} = 313 \text{ m/s}$$

$\omega = 2\pi\nu = 2 \times 3.14 \times 512 = 3215.36 \text{ rad/s}$

$$\nu = \nu_\lambda \Rightarrow \lambda = \frac{V}{\nu} = \frac{313}{512} = 0.611 \text{ m}$$

Propagation constant $k = \frac{2\pi}{\lambda} = \frac{2 \times 3.14}{0.611}$

$= 10.27 \text{ m}^{-1}$

The equation of the wave is

$$\nu(x, t) = A \sin(\omega t - kx)$$

$$= 0.1 \sin(3215.3t - 10.27x)$$

Here x, y are in meter and t is in second

25. A steel wire 0.84 m long has a mass $1.68 \times 10^{-3} \text{ kg}$. If the wire is under a tension 80 N , what is the speed of transverse waves on the wire?

Ans: 200 m/s

Sol: $\mu = \frac{m}{l}$

$$= \frac{1.68 \times 10^{-3} \text{ kg}}{0.84 \text{ m}} = 2 \times 10^{-3} \text{ kgm}^{-1}$$

$T = 80 \text{ N}$

Speed of wave on the wire is given by

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80}{2 \times 10^{-3} \text{ kg/m}}} = \sqrt{40,000}$$

$$= 200 \text{ m/s}$$

26. Estimate the speed of sound in air at standard temperature and pressure. The mass of 1 mole of air is $29 \times 10^{-3} \text{ kg}$

Ans: 331 m/s

Sol: we know that 1 mole of any gas occupies 22.4 litres at STP

Density of air at STP is

$$\rho_0 = \frac{\text{mass of one mole of air}}{\text{volume of one mole of air at STP}}$$

$$= \frac{29 \times 10^{-3} \text{ kg}}{22.4 \times 10^{-3} \text{ m}^3}$$

$$= 1.29 \text{ kgm}^{-3}$$

According to Newton's formula for the speed of sound in a medium. we get for the speed of sound in air at STP

$$v = \sqrt{\frac{P}{\rho}} = \left(\frac{1.01 \times 10^5 \text{ N/m}^2}{1.29 \text{ kgm}^{-3}} \right)^{1/2}$$

$$= (7830)^{1/2}$$

$$= 280 \text{ m/s}$$

27. Two sound sources produce progressive waves given by $y_1 = 12 \cos 100\pi t$ & $y_2 = 12 \cos 102\pi t$ near the ear of an observer. when sounded together, the observer will hear beats. The observed beat frequency (in Hz) is _____

Ans: 1 Hz

Sol: frequency of two source

$$f_1 = 50 \text{ Hz}, \quad f_2 = 51 \text{ Hz}$$

Beat frequency = $|f_2 - f_1| = 1 \text{ Hz}$

28. A resonance tube is resonated with tuning fork of frequency 256 Hz. If the length of first and second resonating air column are 32 cm & 100 cm, then end correction will be _____ cm

Ans: e = 2 cm

$$\text{Sol: } \frac{\lambda}{4} = l_1 + e \text{ ---- (1)}$$

$$\frac{3\lambda}{4} = l_2 + e \text{ ---- (2)}$$

From (1) & (2) e = 2 cm

29. The percentage change in the tension necessary in a sonometer of fixed length to produce a note one octave lower (half of original frequency) than before is _____ %

Ans: 75%

Sol: $v \propto \sqrt{T}$

$$\frac{v_1}{v_2} = 2 = \sqrt{\frac{T_1}{T_2}}$$

$$T_2 = \frac{T_1}{4}$$

$$\frac{T_1 - T_2}{T_1} \times 100 = \frac{T_1 - \frac{T_1}{4}}{T_1} \times 100 = 75\%$$

30. A long rope with a mass of 0.20 kg/m is kept under a tension of 80N. If one end oscillates at rate of 4.0 Hz with an amplitude of 15cm, the energy transmitted per second along the rope is _____w

Ans: 28W

Sol: $v = \sqrt{\frac{T}{\mu}} = 20 \text{ m/s}, p = \frac{1}{2} \mu \omega^2 A^2 v = 28 \text{ W}$
