SRIGAYATRI EDUCATIONAL INSTITUTIONS

INDIA

OSCILLATIONS (UT-5 QB)

PHYSICS

1. **A simple harmonic oscillator of angular frequency 2 rad s^{-1} is acted upon by an external force F=sint N. If the oscillator is at rest in its equilibrium position at t=0, its position at later times is proportional to :**

1) $\sin t+\frac{1}{2}\cos 2t$ 2) $\cos t-\frac{1}{2}\sin 2t$ 3) $\sin t-\frac{1}{2}\sin 2t$ 4) $\sin t+\frac{1}{2}\sin 2t$

2. **A particle executes simple harmonic motion and is located at x=a, b and c at times t_0, $2t_0$ and 3 t_0 respectively. The frequency of the oscillation is …**

1) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$ 2) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{3c}\right)$ 3) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$ 4) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

3. **A damped harmonic oscillator has a frequency are 5 osci**llations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to 1/1000 of the original amplitude is close to ------

1) 100s 2) 10s 3) 20s 4) 50 s

4. **A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is :**

1) $\frac{1}{\sqrt{2}}$ 2) 1 3) $\frac{1}{2}$ 4) $\sqrt{2}$

5. **The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be**

1) $\frac{2}{\sqrt{3}}s$ 2) $\frac{3}{2}s$ 3) $\frac{\sqrt{3}}{2}s$ 4) $2\sqrt{3}s$

6. **A simple harmonic motion is represented by** $y=5(\sin 3\pi t+\sqrt{3}\cos 3\pi t)$cm. **The amplitude and time period of the motion are:**

1) $5cm,\frac{2}{3}s$ 2) $10cm,\frac{3}{2}s$ 3) $10cm,\frac{2}{3}s$ 4) $5cm,\frac{3}{2}s$

7. **A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to 1/1000 of the original amplitude is close to:**

1)50 s 2) 10 s 3)100 s 4) 20 s

8. **The displacement of a damped harmonic oscillator is given by** $x(t)=e^{-0.1t}\cos(10\pi t+\varphi)$ **Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to:**

1)7 s 2) 13 s 3)27 s 4) 4 s

9. **A spring whose upstretched length is *l* has a force constant k. The spring is cut into two pieces of upstretched lengths l_1 and l_2 where, l_1 = n l_2 and n is an integer. The ratio k_1 / k_2 of the corresponding force constants k_1 and k_2 will be :**

1) n^2 2) $\frac{1}{n}$ 3) $\frac{1}{n^2}$ 4) n

10. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other?(Mole wt. of silver =108 and Avogadro number =6.02×10^{23} gm mole^{-1})

1) 7.1 N/m 2) 2.2 N/m 3) 5.5 N/m 4) 6.4 N/m

11. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion $10s^{-1}$. At, t=0 the displacement is 5 m What is the maximum acceleration? The initial phase is $\frac{\pi}{4}$.

1) $500\sqrt{2}m/s^2$ 2) $500m/s^2$ 3) $750\sqrt{2}m/s^2$ 4) $750m/s^2$

12. A block of mass 0.1 kg is connected to an elastic spring of spring constant 640 Nm^{-1} and oscillates in a damping medium of damping constant $10^{-2}kgs^{-1}$. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to :

1) 7s 2) 5s 3) 3.5 s 4) 2 s

13. A particle moves with simple harmonic motion in a straight line. In first τS after starting from rest it travels a distance a, and in next τS it travels 2 a, in same direction, then:

1) amplitude of motion is 3 a 2) time period of oscillations is 8τ

3) amplitude of motion is 4 a 4) time period of oscillations is 6τ

14. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in second is

1) $\frac{8\pi}{3}\ \frac{1}{2}$ 2) $\frac{4\pi}{3}$ 3) $\frac{3\pi}{8}$. 4) $\frac{7\pi}{3}$

15. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be

1) $2\sqrt{3}s$ 2) $\frac{3}{2}s$ 3) $\frac{2}{\sqrt{3}}s$ 4) $\frac{\sqrt{3}}{2}s$

16. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2}m. The relative change in the angular frequency of the pendulum is best given by

1) $10^{-5}rad/s$ 2) $10^{-1}rad/s$ 3) 1 rad/s 4) $10^{-3}rad/s$.

17. Two simple harmonic motions, as shown below, are at right angles. They are combined to form Lissajous figures. $x(t)=A\sin(at+\delta)$ $y(t)=B\sin(bt)$ Identify the correct match below.

Parameters Curve

1) $A=B, a=b; \delta=\frac{\pi}{2}$ Line 2) $A\neq B, a=b; \delta=0$ Parabola

3) $A=B, a=2b; \delta=\frac{\pi}{2}$ Circle 4) $A\neq B, a=b; \delta=\frac{\pi}{2}$ Ellipse

18. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is 1/16th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

1) $4T\sqrt{\frac{1}{15}}$ 2) $2T\sqrt{\frac{1}{10}}$ 3) $4T\sqrt{\frac{1}{14}}$ 4) $2T\sqrt{\frac{1}{14}}$

19. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of $10^{12}s^{-1}$. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver =108 and Avogadro number ….= 6.02 $\times10^{23}$gm mole^{-1})

1) 6.4 Nm^{-1} 2) 7.1 Nm^{-1} 3) 2.2 Nm^{-1} 4) 5.5 nm^{-1}

20. A simple pendulum, made of a string of length *l* and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by:

1) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ 2) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$ 3) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ 4) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

21. The resultant amplitude due to super position of two SHM is
$x_1 = 10\sin(\omega t+30)\text{cm}, x_2 = -10\cos(\omega t+60)\text{cm}$ in **…**

22. A particle executes SHM along a straight line with mean position x = 0 time period 20s and Amplitude 5 cm. The shortest time (in s) taken by the particle to go from x = 4 cm to x = -3cm is ….

23. If the increases in length of a simple pendulum is 5% and increase in acceleration due to greater is 1% then the increases in time period of the pendulum will be ……%

24. A spherical solid ball of radius r makes small oscillations on a rough concave of large radius 'B'. If its rolls on the surface during its oscillation. Its time period is $2\pi\sqrt{\frac{x(R-r)}{5g}}$ find value of x …………m

25. a force of 6.4 N stretches a vertical spring by 0.1m. The mass that it oscillated with a period of (π/4) seconds is …………….

26. A particle located at x = a at time t = 0 starts moving along the positive x-direction with a velocity 'V' that various as $V=\alpha\sqrt{x}$, the displacement of the particle varies with time as t^n. Find the value n is (α = constant)

27. The amplitude of a damped oscillator half in one minute. The amplitude after 3 minutes will be 1/x times the original value. Then x is

28. A particle starting from mean position executes SHM with period 8s. The minimum time in which units P.E. becomes half of the total energy is ……sec.

29. When the length of a simple pendulum is increased by 36cm. its time period of oscillation is found to increase by 25%. The initial length of the simple pendulum is (g = 9.8 ms^2) 8x cm. Then x is …..

30. Three masses 0.1 kg, 0.3 kg and 0.4 kg are suspended at the end of the spring. When the 0.4 Kg is removed the system oscillated with a period of 2 sec. when 0.3 kg mass is also removed the system will oscillated with period is …. sec.

KEY

PHYSICS

1) **3**	2) **4**	3) **3**	4) **1**	5) **4**	6) **3**	7) **4**	8) **1**	9) **2**	10) **1**
11) **1**	12) **3**	13) 0.5	14) **1**	15) **1**	16) **4**	17) **4**	18) **1**	19) **2**	20) **2**
21) **0**	22) **2%**	23) **5**	24) **7**	25) **1**	26) **2**	27) **8**	28) **1**	29) **8**	30) **1**

SOLUTIONS

1.

As we know,

$F = ma \Rightarrow a \propto F$

or, $a \propto \sin t$

$\Rightarrow \frac{dv}{dt} \propto \sin t$

$\Rightarrow \int_0^0 dV \propto \int_0^t \sin t dt$

$V \propto -\cos t + 1$

$\int_0^x dx = \int_0^t (-\cos t + 1) dt$

$x = \sin t - \frac{1}{2} \sin 2t$

2. Using $\mathrm{y} = \mathrm{A} \sin \omega$

$a = A \sin \omega t_0$

$b = A \sin 2\omega t_0$

$c = A \sin 3\omega t_0$

$a + c = A\left[\sin \omega t_0 + A \sin 3\omega t_0\right] = 2A \sin 2\omega t_0 \cos \omega t_0$

$\frac{a+c}{b} = 2\cos \omega t_0$

$\Rightarrow \omega = \frac{1}{t_0} \cos^{-1}\left(\frac{a+c}{2b}\right) \Rightarrow f = \frac{1}{2\pi t_0} \cos^{-1}\left(\frac{a+c}{2b}\right)$

3. **Frequency** of damped oscillation , $\upsilon = 5Hz$

For $A = \frac{A}{2}, t_1 = 2s$

$Also, A = A_0 e^{-\frac{b}{2m}t}$

$\frac{b}{m} = \log 2$

$For A = \frac{A}{1000}, t_2 = ?$

$\frac{1}{1000} = e^{\frac{-b}{2m}t_2}$

$\frac{b}{2m} t_2 = 3\log 10$

$t_2 = \frac{6\log 10}{\log 2}$

$t_2 = 20 \sec$

4. Potential energy of spring $= \frac{1}{2} kx^2$

Here, $x =$ distance of block from mean position,

$k =$ spring constant

At mean position, potential energy $= \frac{1}{2} kA^2$

At equilibrium position, half of the mass of block breaks off, so its potential energy becomes half.

Remaining energy $= \frac{1}{2}\left(\frac{1}{2}kA^2\right) = \frac{1}{2}kA'^2$

Here, A'= New distance of block from mean position

5. $T = 2\pi\sqrt{\frac{l}{g}}, \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}, \quad T_2 = 2\sqrt{\frac{g_1}{g_2}}$

$g_2 = \frac{3GM}{(3R)^2} = \frac{g}{3}, T_2 = 2\sqrt{3}s$

6. $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

$y = 10\sin\left(3\pi t + \frac{\pi}{3}\right), \quad A = 10\text{cm}$

$\omega = 3\pi, T = \frac{2\pi}{\omega} = \frac{2}{3}\text{s}$

7. $A = \frac{A_0}{2}$ after10 oscillations

$\therefore$ After2 seconds

$\frac{A_0}{2} = A_0 e^{-\gamma(2)}, \quad 2 = e^{2\gamma}$

$\ln 2 = 2\gamma, \quad \gamma = \frac{\ln 2}{2}$

$\because A = A_0 e^{-\gamma t}, \quad \ln\frac{A_0}{A} = \gamma t$

$\ln 1000 = \frac{\ln 2}{2}t, \quad 2\left(\frac{3\ln 10}{\ln 2}\right) = t$

$\frac{6\ln 10}{\ln 2} = t, t = 19.931\text{sec}, \quad t \approx 20\text{sec}$

8. $A = A_0 e^{-0.1t} = \frac{A_0}{2}$

$\ln 2 = 0.1t, \quad t = 10\ln 2 = 6.93 \approx 7\text{sec}$

9. $k_1 = \frac{C}{\ell_1}, \quad k_2 = \frac{C}{\ell_2}$

$\frac{k_1}{k_2} = \frac{C\ell_2}{\ell_1 C}\ell_2 = \frac{\ell_2}{n\ell_2} = \frac{1}{n}$

10. $f = 10^{12}\text{Hz}, \quad F = Kx\left[\because K = m\omega^2\right]$

$K = \frac{M}{N}\times(2\pi f)^2$

$= \frac{108\times10^{-3}}{6.023\times10^{23}}\times4\times(3.14)^2\times10^{24}$

$= 7.1\text{N/m}$

11. $a_{max} = A\omega^2, v_{max} = A\omega$

$$\frac{a_{max}}{v_{max}} = \omega = 10, \quad x = A\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$\text{at } t = 0, \quad 5 = A\sin\frac{\pi}{4} \Rightarrow A = 5\sqrt{2}$$

$$a_{max} = A\omega^2 = 5\sqrt{2} \times 100 = 500\sqrt{2}\text{m/s}^2$$

12. $E' = \frac{1}{2} \quad b = \frac{\lambda}{m} A = A_0 e^{-bt}$

$$a' = \frac{a}{\sqrt{2}} \lambda = bm\frac{1}{\sqrt{2}} = e^{\frac{t}{10}}$$

$$b = \frac{1}{10} \quad \sqrt{2} = e^{\frac{1}{10}}, \ln\sqrt{2} = \frac{t}{10}, \quad t = 3.5$$

13. $\text{from}(2)\cos 2\omega\tau = 1 - \frac{3a}{A}$

$$but \cos 2\omega\tau = 2\cos^2\omega\tau - 1$$

$$1 - \frac{3a}{A} = 2\left(1 - \frac{a}{A}\right)^2 - 1$$

$$1 - \frac{3a}{A} = 2\left(1 + \frac{a^2}{A^2} - \frac{2a}{A}\right) - 1$$

$$\frac{2a^2}{A^2} = \frac{a}{A} \Rightarrow a = \frac{A}{2}, \quad A = 2a$$

$$A = 2asubin(1)$$

$$a = 2a(1 - \cos\omega\tau)$$

$$1 - \cos\omega\tau = \frac{1}{2}$$

$$\cos\omega\tau = \frac{1}{2}$$

14. In SHM, speed, $v = \omega\sqrt{A^2 - x^2}$

Acceleration, $a = -\omega^2 x$

As $|v| = |a|$

$$\Rightarrow \quad \omega\sqrt{A^2 - x^2} = \omega^2 x \Rightarrow A^2 - x^2 = \omega^2 x^2 \Rightarrow \omega^2 = \frac{A^2 - x^2}{x^2}$$

$$= \frac{5^2 - 4^2}{4^2} = \left(\frac{3}{4}\right)^2 \Rightarrow \omega = \frac{3}{4} \Rightarrow T = \frac{2\pi}{\omega} = \frac{8\pi}{3}$$

15. Time period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{1}{g}}$$

$$g_p = \frac{GM_p}{R_p^2} = 4\frac{GM_p}{D_p^2} = \frac{(4GM_E)}{D_E^2} \times \frac{3}{3^2}$$

$$g_p = \frac{g_e}{3} \qquad \left(\because g_e = \frac{4GM_B}{D_E^2}\right)$$

$$\therefore \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} = \frac{1}{\sqrt{3}}; T_p = 2\sqrt{3}\text{s} \qquad (\because T_e = 2s)$$

16. Angular frequency of the pendulum.

$$\omega=\sqrt{\frac{g_{\text{eff}}}{l}},\frac{\Delta\omega}{\omega}=\frac{1}{2}\frac{\Delta g_{\text{eff}}}{g_{\text{eff}}}$$

$$\Delta(0)=\frac{12A\omega_s^2}{2}\omega=\frac{1}{2}\times\frac{2\times(1)^2\times(10)^{-2}}{10}=10^{-3}\text{rad/s}$$

17. 1 - Circle, 2 - Straight line
3 - Parabola, 4 - Ellipse

$$x=A\sin\left(at+\frac{\pi}{2}\right)$$

$$x=A\cos at\Rightarrow\cos at=\frac{x}{A}$$

$$y=B\sin at\Rightarrow\sin at=\frac{y}{B}$$

$$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1\Rightarrow\text{ Ellipse}$$

18. For air $g=g,T=2\pi\sqrt{\frac{l}{g}}$

For liquid

$$g'=g\left[1-\frac{\rho_{\text{liquid}}}{\rho_{\text{body}}}\right]=g\left[1-\frac{1}{16}\right]=\frac{15}{16}g$$

$$T'=2\pi\sqrt{\frac{l}{\left(\frac{15}{16}\right)g}}$$

From equation (i) and (ii)

$$\frac{T'}{T}=\frac{2\pi\sqrt{l/(15/16)g}}{2\pi\sqrt{l/g}}\Rightarrow T'=\frac{4T}{\sqrt{15}}$$

19. Frequency of a particle executing SHM,

$$v=\frac{1}{2\pi}\sqrt{\frac{k}{m}};k=4\pi^2\times v^2\times m$$

Here, $v=10^{12}\text{s}^{-1},m=\frac{108}{6.02\times10^{23}}\times10^{-3}\text{kg},k=?$

$$\therefore k=4\times(3.14)^2\times\left(10^{12}\right)^2\times\frac{108\times10^{-3}}{6.02\times10^{23}}=7.1\text{Nm}^{-1}$$

20. $\text{u}=\omega\theta_0,\quad \text{v}=\omega\theta_1$

$$\Rightarrow\frac{u}{v}=\frac{\theta_0}{\theta_1},\text{ Now, }v=\frac{M-m}{M+m}\times u$$

$$\Rightarrow\frac{M+m}{M-m}=\frac{u}{v}=\frac{\theta_0}{\theta_1}\Rightarrow\frac{M}{m}=\frac{\theta_0+\theta_1}{\theta_0-\theta_1}$$

$$\Rightarrow M=m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$$

21. $x_1=10\sin(\omega t+30)$

$x_2=-10\cos(\omega t-30)$

$x=10\sin(270+\omega t-60)$

$x_2 = 10\sin(\omega t + 210)$

$\phi = 210 - 30$

$\phi = 180^0$

$A : \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos 180}$

$A = A_1 - A_2$

$A = 10 - 10 = 0$

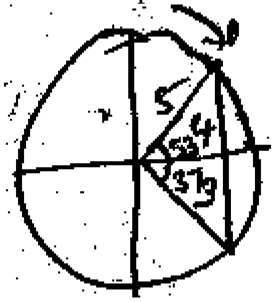
22. $\frac{\Delta l}{l} \times 100 = 5\%$

$\frac{\Delta g}{g} \times 100 = 1\%$

$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\left(\frac{\Delta l}{l} \times 100 - \frac{\Delta g}{g} \times 100\right)$

$\frac{\Delta T}{T} \times 100 = \frac{1}{2}(5-1) = \frac{4}{2} = 2\%$

23. minimum phase difference before two position



$\therefore 53 + 37 = 90$

Time taken $= T/4 = \frac{20}{4}$

$T = 5$ see

24. $T = 2\pi\sqrt{\frac{(R - r')}{g}(1+\beta)}$

$\beta = \frac{2}{5}$ (solid boll)

$T = 2\pi\sqrt{\frac{(R-r)}{g}\left(1 + \frac{2}{5}\right)} = 2\pi\sqrt{\frac{7}{5}\frac{(R-r)}{g}}$

$x = 7m$

25. $T = 2\pi\sqrt{\frac{m}{k}}$

$T = \frac{\pi}{4}$ sec

$F = kx$

$\text{k} = \frac{F}{x} = \frac{6.4}{0.1} = 64\text{N/m}$

$\frac{\pi}{4} = 2\pi\sqrt{\frac{m}{8}}$

$\frac{1}{4}=\frac{2}{8}\sqrt{m} \Rightarrow m=1kg$

m= 1 KG

26. $v=\alpha\sqrt{x}$

$x \;\alpha\; t^2$

$\frac{dx}{dr}=\alpha\sqrt{x}$

$\frac{dx}{\sqrt{x}}=\alpha dr$

$n=2$

27. Amplatuded dampet os cilload $A=A_0e^{-\lambda t}$

for $t=1\text{min}$ $\frac{A_2}{2}=A_0e^{-\lambda(t)} \Rightarrow e^{\lambda}=2$

For t = 3 min $A=A_1e^{-3\lambda} \Rightarrow \frac{A}{(e^{\lambda})^3}=\frac{A}{2^3}$

$\therefore x=2^3=\underline{8m}$

28. T = 8s

$U=\frac{T.E}{2}$

$\frac{1}{2}mw^2y^2=\frac{\frac{1}{2}m\omega^2A}{2}$

$A\sin\omega t=\frac{A}{\sqrt{2}} \Rightarrow \sin\omega t=\frac{1}{\sqrt{2}}$

$y=\frac{A}{\sqrt{2}}$

$y=$ Asinwt

$y^2=\frac{A^2}{2}$

$\frac{2\pi}{2}t=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \frac{2\pi}{t}t=\frac{\pi}{4}$

$t=\frac{T}{8}=\frac{8}{8}=1\sec$

29. $T=2\pi\sqrt{\frac{l}{g}}$

$\frac{T_1}{T_2}=\sqrt{\frac{l_1}{l_2}}$

$$\left(\frac{100}{125}\right)^2 = \frac{l_1}{l_1+36}$$

$$\frac{16}{25} = \frac{l_1}{l_1+36}$$

$$l_1 = 64\text{cm}$$

$$l_1 = 8\times 8cm$$

$$x = 8$$

30. $T=2\pi\sqrt{\frac{M}{k}}$

0.4 Kg removed remain load = 0.1+0.3 = 0.4

$$2=2\pi\sqrt{\frac{0.4}{k}} \rightarrow (1)$$

0.3 Kg also removed remain load – 0.1 Kg

$$T^1=2\pi\sqrt{\frac{0.1}{k}} \rightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{2}{T^1} = \sqrt{\frac{0.4}{0.1}} \Rightarrow \frac{2}{T'} = 2$$

$$T' = 1\text{sec}$$